Yang-Mills propagators close to the phase transition

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Strong Interactions in Quantum Field Theory

Introduction

- We study Yang-Mills (YM) theories at finite temperature close to the phase transition.
- We use an analytic (perturbative) approach in order to access the Green functions.
- Our approach is motivated by previous studies at zero temperature, where the deep infrared regime was probed with simple perturbation theory.

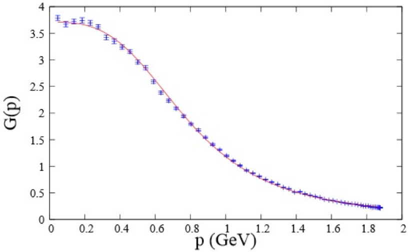
Preliminary: vacuum study

- Gauge-fixed lattice simulations (Landau gauge) have shown that gluons are massive.
- We consider an effective massive theory to reproduce lattice results: Curci-Ferrari (CF) model. [G. Curci, R. Ferrari, Nuovo Cim. A 32, (1976)]

CF MODEL

- Infrared safe RG trajectories down to zero momentum.
- One-loop calculations are in very good agreement with lattice results.

[M. Tissier, N. Wschebor, Phys. Rev. D84 (2011)]



- The effective mass naturally arises in a new quantization scheme which consistently deals with Gribov ambiguities in Landau gauge. [J. Serreau, M. Tissier, Phys. Lett. B712 (2012)]
- Most of the gluonic non-perturbative dynamics is accurately captured by the effective mass. [M. Peláez, M. Tissier, N. Wschebor, Phys. Rev. D 88 (2013)]
- Does a similar massive approach allow for a perturbative description of YM correlators at finite temperature around the phase transition?

Preliminary: massive thermal YM

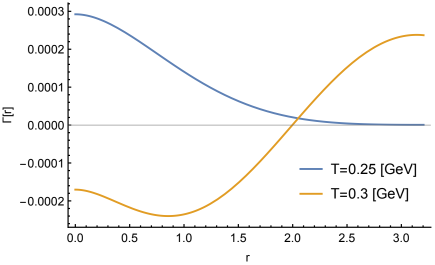
Massive Landau gauge at one-loop order: $SU(2)$

[U. Reinosa, J. Serreau, M. Tissier, N. Wschebor, Phys. Rev. D89 (2014)]

- Ghost and magnetic gluon in agreement with lattice simulations.
- Electric gluon close to T_c does not describe well the lattice results.

Order parameter for the phase transition: $\bar{A}_\mu$, background field

- Analogy with mean field theory of the Ising model.
- Perturbation theory around the background: $a_\mu = A_\mu - \bar{A}_\mu$.



Two-loop potential $\Gamma[\bar{A}]$ in $SU(2)$.

[U. Reinosa, J. Serreau , M. Tissier , N. Wschebor, Phys. Rev. D91 (2015)]

- $\Gamma[\bar{A}]$ is minimum for $\bar{A} = \frac{rT}{g} \neq 0$.
- Landau $(\bar{A} = 0)$ is a maximum!
- Phase transition is observed perturbatively.

The Landau-DeWitt gauge

Background field methods to keep track of the relevant (center) symmetry

[J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B 684 (2010)]

- Gauge condition $\underbrace{\partial_\mu a_\mu - ig[\bar{\mathbf{A}}_\mu, a_\mu]}_{\bar{D}_\mu a_\mu} = 0.$
- $SU(2)$ gauge group with $\bar{A}_\mu = \bar{A}\delta_{\mu 0}\delta^{3a}t^a \to$ one color direction is singled out.
- $\Rightarrow$ No more diagonal matrix form (the usual δ^{ab} structure) in the basis $\{t^a\}$.

Color charge κ and canonical (Weyl) basis $\{t^\kappa\}$

- $\{t^a\} \to \{t^\kappa\} = \{t^0, t^+, t^-\}$, (like for spin).
- $\bar{D}_\mu$ diagonal in this basis: $\bar{D}_\mu \to \partial^\kappa_\mu = \partial_\mu + g\kappa\delta_{\mu 0}\bar{A}$.

In Fourier space: $\partial^\kappa_\mu \to i\left(K_\mu + g\kappa\bar{A}\delta_{\mu 0}\right) \Rightarrow$ shifts the Matsubara frequencies by $g\kappa\bar{A}$.
Define generalized momentum:

$$K^\kappa_\mu = K_\mu + g\kappa\bar{A}\delta_{\mu 0}$$

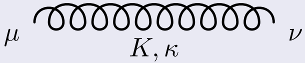
Massive Landau-DeWitt action

$$S_{\bar{A}} = \int_x \mathrm{tr}\left\{\frac{1}{2}F_{\mu\nu}F_{\mu\nu} + \mathbf{m^2} a_\mu a_\mu + 2\bar{D}_\mu \bar{c} D_\mu c + 2ih\bar{D}_\mu a_\mu\right\}.$$

- Mass term introduced such as it does not break the background field gauge symmetry:
  $S_{\bar{A}} = S_{\bar{A}^U}$, $\bar{A}^U$ gauge transformed of $\bar{A}$.
  This insures that the center symmetry is not explicitly broken.

Propagators

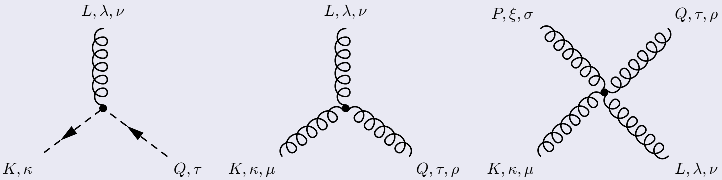
$$G^{\kappa}_{gh}(K) = \frac{1}{(K^\kappa)^2}$$



$$G^{\kappa}_{\mu\nu}(K) = \frac{P^{\perp}_{\mu\nu}(K^\kappa)}{\mathbf{m^2} + (K^\kappa)^2}$$

Feynman rules

Interaction vertices



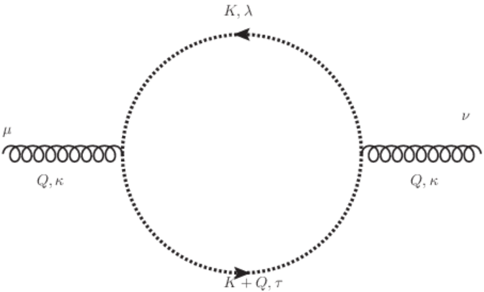
- E.g. ghost-antighost-gluon vertex: $g f^{\kappa\lambda\tau} K^{\kappa}_{\mu}$.

Color charge conservation

$\Rightarrow$ Shift conservation at vertices: $\kappa + \tau + \lambda = 0$.
$\Rightarrow$ Conservation of generalized momenta $K^{\kappa} + Q^{\tau} + L^{\lambda} = 0$.

In the $SU(2)$ case, the structure constant is $\epsilon^{0+-} = 1$.

One-loop computations



$$\Pi^{gh}_{\mu\nu}\left(Q^\kappa\right) = \sum\!\!\!\!\!\!\!\int_K \sum_{\lambda,\tau=\{0,+,-\}} f^{\tau(-\kappa)(-\lambda)} f^{\lambda\kappa(-\tau)} (K+Q)^\tau_\mu K^\lambda_\nu \frac{1}{\left((K+Q)^\tau\right)^2 \left(K^\lambda\right)^2}$$

$$= \sum\!\!\!\!\!\!\!\int_K \sum_{\lambda=\{0,+,-\}} \frac{\left|f^{\lambda\kappa(-\kappa-\lambda)}\right|^2 (K^\lambda + Q^\kappa)_\mu K^\lambda_\nu}{\left(K^\lambda + Q^\kappa\right)^2 \left(K^\lambda\right)^2}.$$

Background field dependence in loop contributions

One-loop expressions are expressed as simple sum-integrals.

- All Matsubara sums (and angular integrals) can be performed analytically.
- Tadpole-like:

$$J^{(m,\kappa)} = \sum\!\!\!\!\!\!\!\int_K \frac{1}{\mathbf{m^2} + (K^\kappa)^2}$$
$$= \int_k \frac{n\left((\epsilon_{\mathbf{m},k} - \mathbf{i}\kappa\mathbf{g}\bar{\mathbf{A}})/T\right) + n\left((\epsilon_{\mathbf{m},k} + \mathbf{i}\kappa\mathbf{g}\bar{\mathbf{A}})/T\right) + 1}{2\epsilon_{\mathbf{m},k}}.$$

$$\epsilon_{\mathbf{m},k} = \sqrt{\mathbf{m^2} + k^2} \qquad n(x) = \frac{1}{e^x - 1}$$

- Low temperature phase $g\bar{A}/T = \pi \Rightarrow n(x - \mathbf{i}\pi) = -\frac{1}{e^x+1}$ behaves like fermions!
- Many other integrals are involved, e.g. Sunset-like:

$$I_{\mu\nu}^{(\alpha,\kappa),(\beta,\lambda)}(Q) = \sum\!\!\!\!\!\!\!\int_K K_\mu^\kappa K_\nu^\kappa \frac{1}{\alpha^2 + (K^\kappa)^2} \frac{1}{\beta^2 + (K^\kappa + Q^\lambda)^2}.$$

- The background field acts as an imaginary chemical potential.

One-loop propagators

$$\Pi^{\kappa}_{\mu\nu}(K) =$$

$$\Sigma^{\kappa}(Q) =$$

$$= \sum_{\tau,\lambda} C_{\kappa\tau\lambda} \frac{\left(m^2 - (Q^\kappa)^2\right)}{4m^2} \left(J^{(m,\lambda)} - J^{(0,\lambda)}\right)$$

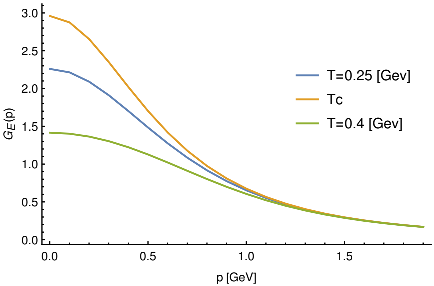
$$+ \frac{\left((Q^\kappa)^2 + m^2\right)^2}{4m^2} I^{(m,\lambda)(0,\tau)} - \frac{(Q^\kappa)^4}{4m^2} I^{(0,\lambda)(0,\tau)}$$

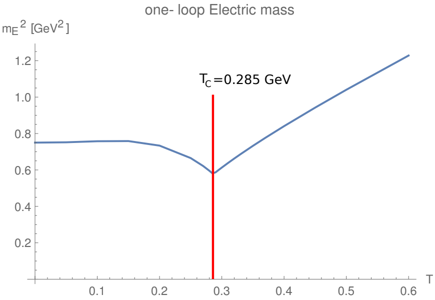
$$+ \frac{\omega^\kappa_Q}{2m^2} \left(\hat{J}^{(m,\lambda)}_0 - \hat{J}^{(0,\lambda)}_0\right).$$

- Smooth limit $\bar{A} \to 0$, $T \to 0$.
- Only vacuum divergences.

Evaluate these contributions at $\bar{A}$ that minimizes $\Gamma[\bar{A}]$ computed at two-loop order.

Preliminary results





- Non-monotonic behavior around T_c (similar to lattice results in the Landau gauge).
- Cusp in the Electric mass at T_c due to the phase transition.

Conclusion and outlook

Massive Landau-DeWitt gauge

1. $\bar{A}$ order parameter for the phase transition.
2. Massive extension of the LDW gauge (mass related to issues of Gribov copies). **[J. Serreau, M. Tissier, Phys. Lett. B712 (2012)], [J. Serreau, M. Tissier, A. Tresmontant Phys. Rev. D89 (2014)], [J. Serreau M. Tissier, A. Tresmontant, appears in Phys. Rev. D, arXiv:1505.07270]**
3. Vacuum study showed renormalizability, asymptotic freedom,...

Up to now

1. Access perturbatively the phase transition (potential Γ up to two loops).
2. Propagators at one loop. Many modes ($\kappa = 0, +, -$) $\rightarrow$ rich phenomenology.

Outlook

1. Lattice simulations in this gauge. Might help for stability issues.
2. $SU(3)$, spectral functions, renormalization group trajectories, Polyakov loop.

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