Real time renormalization

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October 8, 2015

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In-out and in-in formalism

- The usual formalism of quantum field theory is based on the vacuum-to-vacuum transition amplitude.
  - $Z[J] = \langle 0_+ | 0_- \rangle = e^{iW[j]} = \int \mathcal{D}\phi e^{i(S+J\phi)}$
- The states $|0_\pm\rangle$ are usually different, we calculate matrix elements of the operator T according to $\langle 0_+ | T | 0_- \rangle$
  - The in-out formalism can provide **transitions amplitudes** between the in-out states.
  - $\langle 0_- | T | 0_- \rangle$, The in-in formalism can provide **expectation values**
- The generating functional for the in-in formalism is

$$e^{iW[J^+,J^-]} = \sum_n \langle 0_- | n \rangle_{J^-} \langle n | 0_- \rangle_{J^+}$$

- The CTP formalism is based on the invariance of the density matrix

$$
\begin{aligned}
Z[J^+,J^-] &= e^{iW[J^+,J^-]} \\
&= \sum_n \langle 0_-|\bar{T}e^{i\int(H-\phi^- J^-)}|n\rangle\langle n|Te^{-i\int(H-\phi^+ J^+)}|0_-\rangle \\
&= \mathrm{Tr}\left[Te^{-i\int(H-\phi^+ J^+)} \underbrace{|0_-\rangle\langle 0_-|}_{\rho(0_-)} \bar{T}e^{i\int(H-\phi^- J^-)}\right] \\
&= \mathrm{Tr}\left[\rho(t)\right] \\
&= \rho_r(t)
\end{aligned}
$$

- we split the field variable as $\phi = \phi_< + \phi_> = \phi_S + \phi_E$
- the RG method can systematically eliminate the environmental degrees of freedom
- the $\rho_r(t)$ resulting reduced density matrix can account for the decoherence

The generating functional for a scalar field is

$$Z[J_+, J_-] = \int \mathcal{D}\phi_+ \mathcal{D}\phi_- e^{i\int_{t_i}^{t_f}(S[\phi_+]+J_+\phi^+ - S^*[\phi_-] - J_-\phi^-)}$$

where in S^* we have $+i\epsilon$. The scalar CTP propagator is

$$\hat{D}_p^{jk} = \begin{pmatrix} \frac{1}{p^2-m^2+i\epsilon} & -2\pi i\delta(p^2-m^2)\Theta(-p^0) \\ -2\pi i\delta(p^2-m^2)\Theta(p^0) & -\frac{1}{p^2-m^2-i\epsilon} \end{pmatrix}.$$

The inverse CTP propagator is

$$\hat{D}_p^{-1jk} = (p^2-m^2)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i\epsilon \begin{pmatrix} 1 & -2\Theta(-p^0) \\ -2\Theta(p^0) & 1 \end{pmatrix}.$$

The single time Wegner-Houghton equation is

$$\dot{V}_k = i\alpha k^{d-1} \int_\omega \ln D^{-1},$$

which can be generalized into a CTP RG equation

$$\dot{V}_k = i\alpha k^{d-1} \int_\omega \mathrm{Tr} \ln \hat{D}^{-1},$$

where we replaced the single time propagator to its CTP version

$$D^{-1} = \omega^2 - k^2 - V_k'' + i\epsilon \;\Rightarrow \hat{D}^{-1} = \begin{pmatrix} \omega^2 - k^2 - V_k^{++} + i\epsilon & -2i\Theta(-\omega)\epsilon - V_k^{+-} \\ -2i\Theta(\omega)\epsilon - V_k^{-+} & -\omega^2 + k^2 - V_k^{--} + i\epsilon \end{pmatrix}.$$

Here

$$V'' \quad \Rightarrow \quad V_k^{ij} = \begin{pmatrix} \frac{\delta^2 V_k}{\delta\phi_+ \delta\phi_+} & \frac{\delta^2 V_k}{\delta\phi_+ \delta\phi_-} \\ \frac{\delta^2 V_k}{\delta\phi_- \delta\phi_+} & \frac{\delta^2 V_k}{\delta\phi_- \delta\phi_-} \end{pmatrix} \equiv \begin{pmatrix} V_k^{++} & V_k^{+-} \\ V_k^{-+} & V_k^{--} \end{pmatrix}$$

ACKNOWLEDGMENTS

The financial support from the Hungarian National Research Fund OTKA (K112233) is acknowledged.

Thank you for your attention!