

Real time renormalization

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- The usual formalism of quantum field theory is based on the vacuum-to-vacuum transition amplitude.
 - $Z[J] = \langle 0_+ | 0_- \rangle = e^{iW[J]} = \int \mathcal{D}\phi e^{i(S+J\phi)}$
- The states $|0_{\pm}\rangle$ are usually different, we calculate matrix elements of the operator T according to $\langle 0_+ | T | 0_- \rangle$
 - The in-out formalism can provide **transitions amplitudes** between the in-out states.
 - $\langle 0_- | T | 0_- \rangle$, The in-in formalism can provide **expectation values**
- The generating functional for the in-in formalism is

$$e^{iW[J^+, J^-]} = \sum_n \langle 0_- | n \rangle_{J^-} \langle n | 0_- \rangle_{J^+}$$



- The CTP formalism is based on the invariance of the density matrix

$$\begin{aligned}
 Z[J^+, J^-] &= e^{iW[J^+, J^-]} \\
 &= \sum_n \langle 0_- | \bar{T} e^{i \int (H - \phi^- J^-)} | n \rangle \langle n | T e^{-i \int (H - \phi^+ J^+)} | 0_- \rangle \\
 &= \text{Tr} \left[T e^{-i \int (H - \phi^+ J^+)} \underbrace{| 0_- \rangle \langle 0_- |}_{\rho(0_-)} \bar{T} e^{i \int (H - \phi^- J^-)} \right] \\
 &= \text{Tr} [\rho(t)] \\
 &= \rho_r(t)
 \end{aligned}$$

- we split the field variable as $\phi = \phi_{<} + \phi_{>} = \phi_S + \phi_E$
- the RG method can systematically eliminate the environmental degrees of freedom
- the $\rho_r(t)$ resulting reduced density matrix can account for the decoherence



The generating functional for a scalar field is

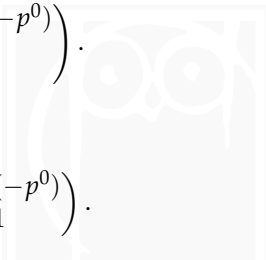
$$Z[J_+, J_-] = \int \mathcal{D}\phi_+ \mathcal{D}\phi_- e^{i \int_{t_i}^{t_f} (S[\phi_+] + J_+ \phi^+ - S^*[\phi_-] - J_- \phi^-)}$$

where in S^* we have $+i\epsilon$. The scalar CTP propagator is

$$\hat{D}_p^{jk} = \begin{pmatrix} \frac{1}{p^2 - m^2 + i\epsilon} & -2\pi i \delta(p^2 - m^2) \Theta(-p^0) \\ -2\pi i \delta(p^2 - m^2) \Theta(p^0) & -\frac{1}{p^2 - m^2 - i\epsilon} \end{pmatrix}.$$

The inverse CTP propagator is

$$\hat{D}_p^{-1jk} = (p^2 - m^2) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i\epsilon \begin{pmatrix} 1 & -2\Theta(-p^0) \\ -2\Theta(p^0) & 1 \end{pmatrix}.$$



The single time Wegner-Houghton equation is

$$\dot{V}_k = i\alpha k^{d-1} \int_{\omega} \ln D^{-1},$$

which can be generalized into a CTP RG equation

$$\dot{V}_k = i\alpha k^{d-1} \int_{\omega} \text{Tr} \ln \hat{D}^{-1},$$

where we replaced the single time propagator to its CTP version

$$D^{-1} = \omega^2 - k^2 - V_k'' + i\epsilon \Rightarrow \hat{D}^{-1} = \begin{pmatrix} \omega^2 - k^2 - V_k^{++} + i\epsilon & -2i\Theta(-\omega)\epsilon - V_k^{+-} \\ -2i\Theta(\omega)\epsilon - V_k^{-+} & -\omega^2 + k^2 - V_k^{--} + i\epsilon \end{pmatrix}.$$

Here

$$V'' \Rightarrow V_k^{ij} = \begin{pmatrix} \frac{\delta^2 V_k}{\delta\phi_+ \delta\phi_+} & \frac{\delta^2 V_k}{\delta\phi_+ \delta\phi_-} \\ \frac{\delta^2 V_k}{\delta\phi_- \delta\phi_+} & \frac{\delta^2 V_k}{\delta\phi_- \delta\phi_-} \end{pmatrix} \equiv \begin{pmatrix} V_k^{++} & V_k^{+-} \\ V_k^{-+} & V_k^{--} \end{pmatrix}$$

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Thank you for your attention!

