# Fluctuations
# in a finite Volume

**Bernd-Jochen Schaefer**

in collaboration with **Ana Juricic**

Der Wissenschaftsfonds.

**Austria**

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

**Germany**

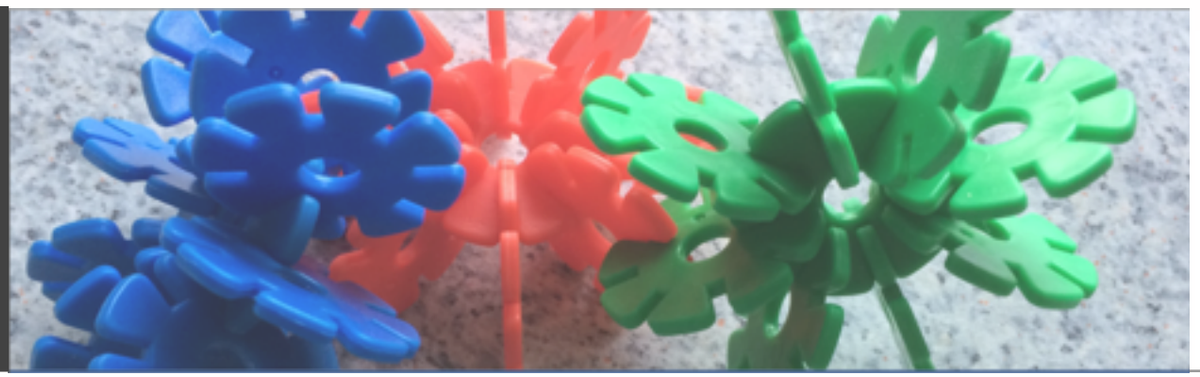
HIC for FAIR
Helmholtz International Center

**Germany**

**October 9th, 2015**

**Leibnitz, Austria**

ACHT2015
Strong Interactions in Quantum Field Theory



7th - 9th October 2015
Leibnitz

UNI GRAZ

Workshop organized by Research Group on Covariant Models of Hadrons

When you tweet about and from the workshop, please use the hashtag #ACHT2015

# Experiments: Heavy-Ion Collision

QCD under extreme conditions: active field for the next 20-25 years ➜ see. e.g. FAIR

One important future HIC Experiment: C(ompressed) B(aryonic) M(atter)

QCD phase diagram and QCD matter Equation of State

<u>in more detail: fundamental phenomena:</u>

- color confinement
- nature of chiral & deconfinement transition
- early Universe history
- nuclear matter
- compact objects (NS,...)
- ...

FAIR construction start 2012

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first good news:

FAIR approved for next 2 years

(2017)

without limitations

➜ bullish mood !

FAIR

***Aug. 2014***

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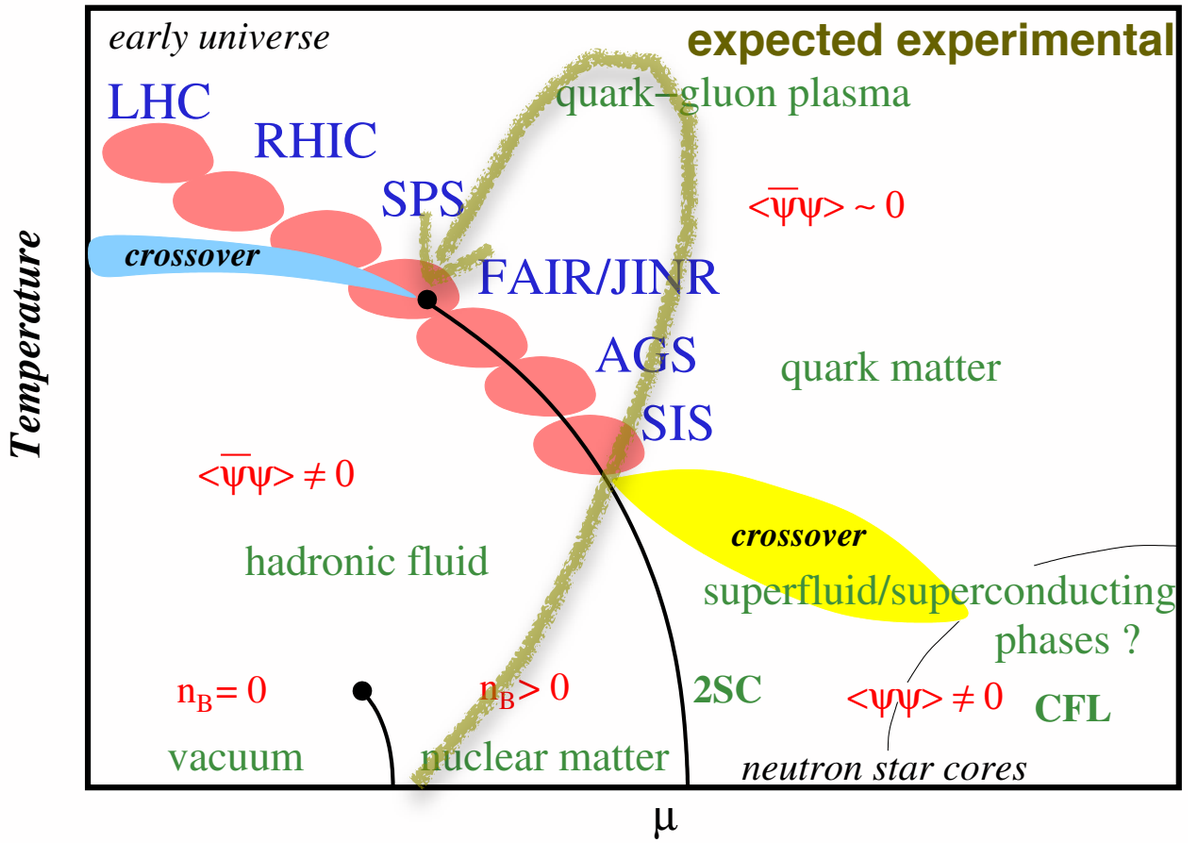
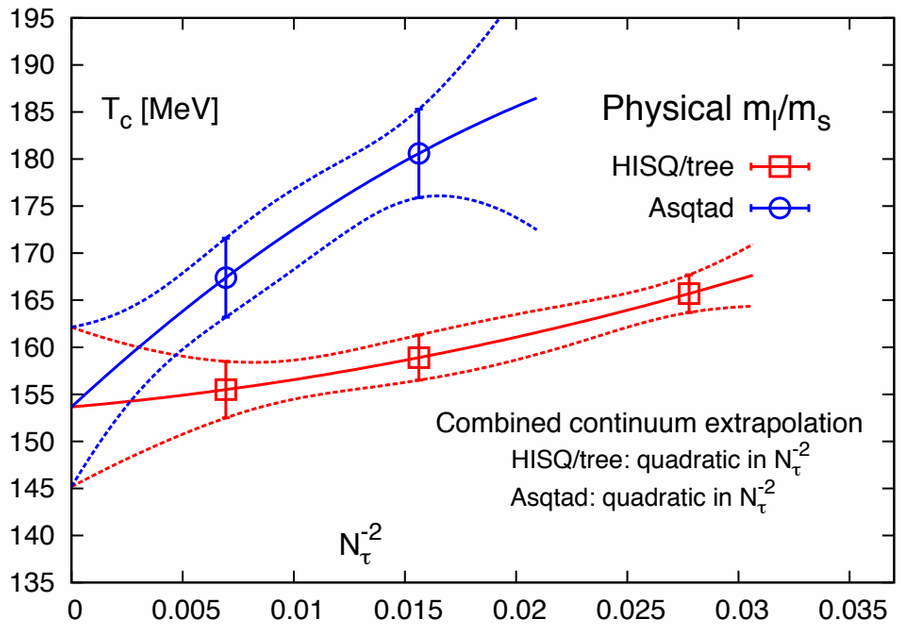
# Conjectured $QC_3D$ phase diagram

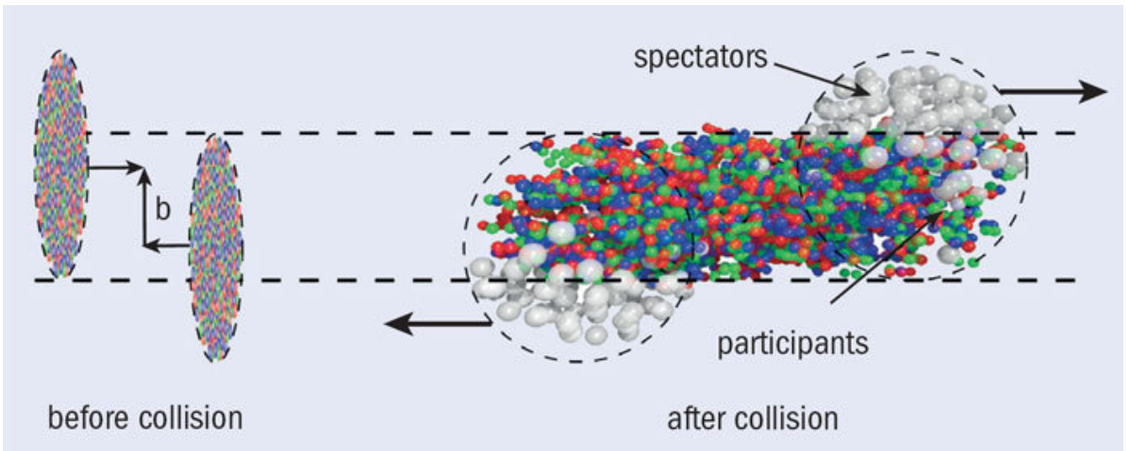



Even QCD lattice simulations not final answer




courtesy of F. Karsch

Experiment:




Theory:

➜ Lattice: but simulations restricted $\mu \simeq 0$

➜ Functional QFT methods: FRG,DSE, nPI

➜ Models: effective theories parameter dependency

# Physics in a finite Volume

Experiment: Heavy-Ion Collision (non-equilibrium) always in a finite box

Theoretical aim: deeper understanding and more realistic HIC description

➜ existence of critical end point(s)?

**Non-trivial physical issues!**

# Physics in a finite Volume

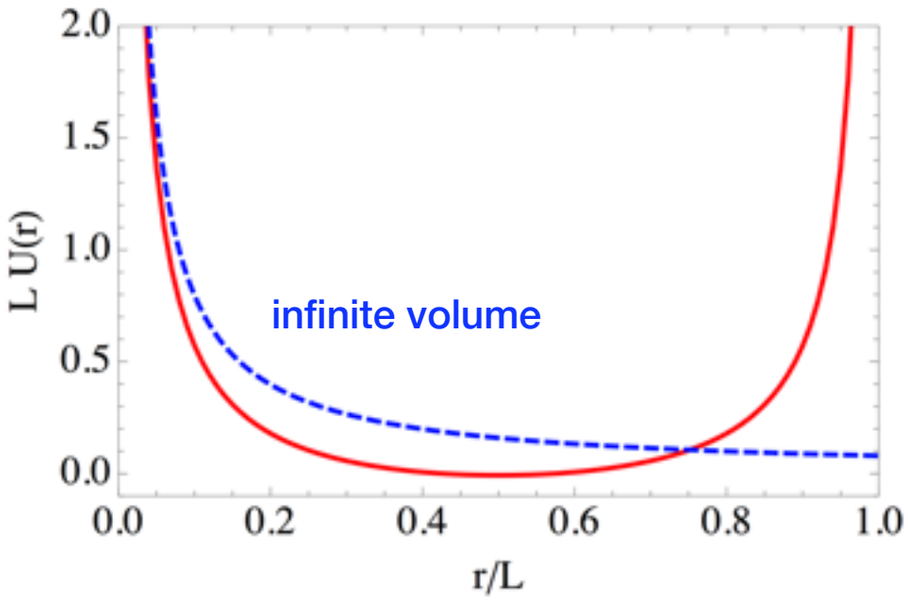
[ Davoudi, Savage 2014]

**Lattice simulations:**

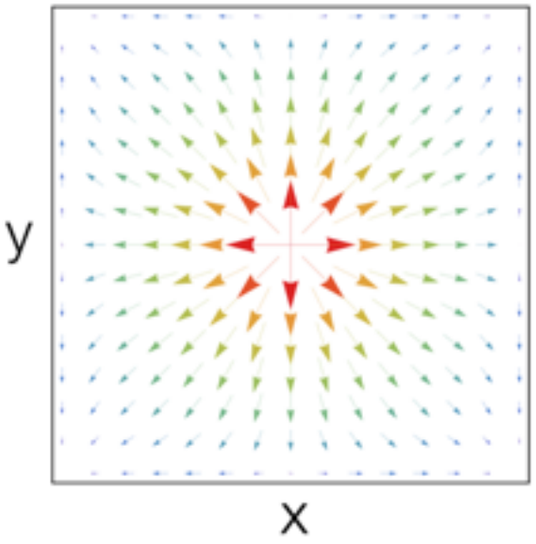
QCD (short-ranged) with QED (long-ranged ➜ truncated) corrections

➜ violation Gauss's & Ampere's law

if EM gauge field subject to periodic boundary condition (PBC)




finite volume Coulomb potential between two charges




point charge at the center

circumvent this problem:

introduce uniform background charge density

similar to many-body physics

➜ equivalent to

removing zero modes of the gauge field

# Physics in a finite Volume

**Quantum Field Theory in a finite volume:**

➜ no spontaneous symmetry breaking

finite number of degrees of freedom

QCD: [ Gasser, Leutwyler 1988]

chiral condensate: a non-perturbative phenomenon

Example:

chiral symmetry

$$N_f = 2 : SU(2) \times SU(2) \cong O(4)$$

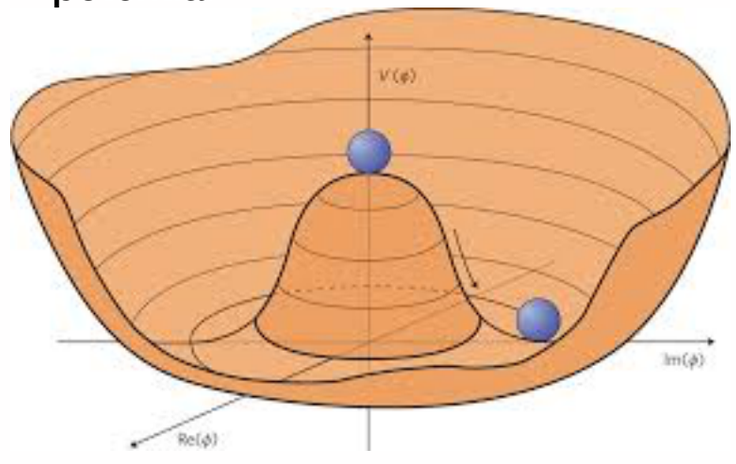
$$O(4) \to O(3)$$ infinite volume

massless Goldstone Bosons

finite volume:

fluctuations of Goldstone bosons always restore symmetry

potential




minimum: zero-momentum mode of the field

$$Z_2 : \varphi \to -\varphi$$

probability of tunneling:

exponentially suppressed by volume

infinite volume ➜ no tunneling (symmetry broken)

$O(N)$ - case: rotation ➜ averaging to zero (no breaking)

# Physics in a finite Volume

result so far:

long-range correlations are necessary to obtain spontaneous SB (for a continuous symmetry)

chiral limit: massless Goldstone boson fluctuations in a finite box avoid symmetry breaking

but

symmetry breaking in mean-field approximations are possible:

Goldstone-fluctuations are absent

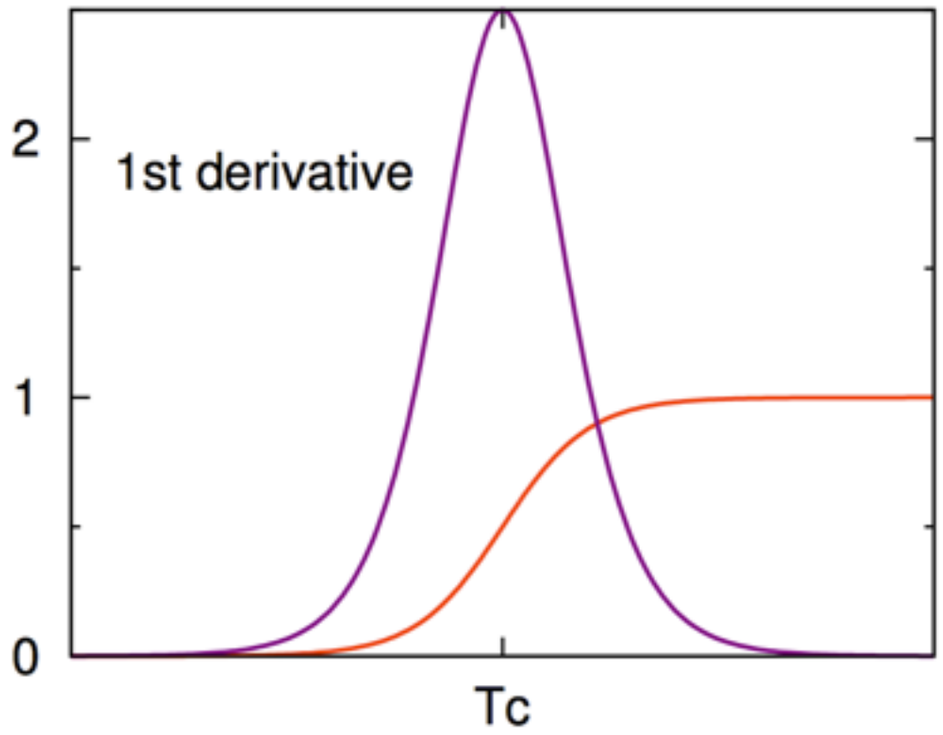
Thermodynamics on a torrus:

correlation length always finite ➜ no real 2nd order phase transition

criterion for phase transition: (generalized) susceptibilities

➜ derivatives of order parameter reveal more details

derivatives of thermodynamic quantities ↔ fluctuations



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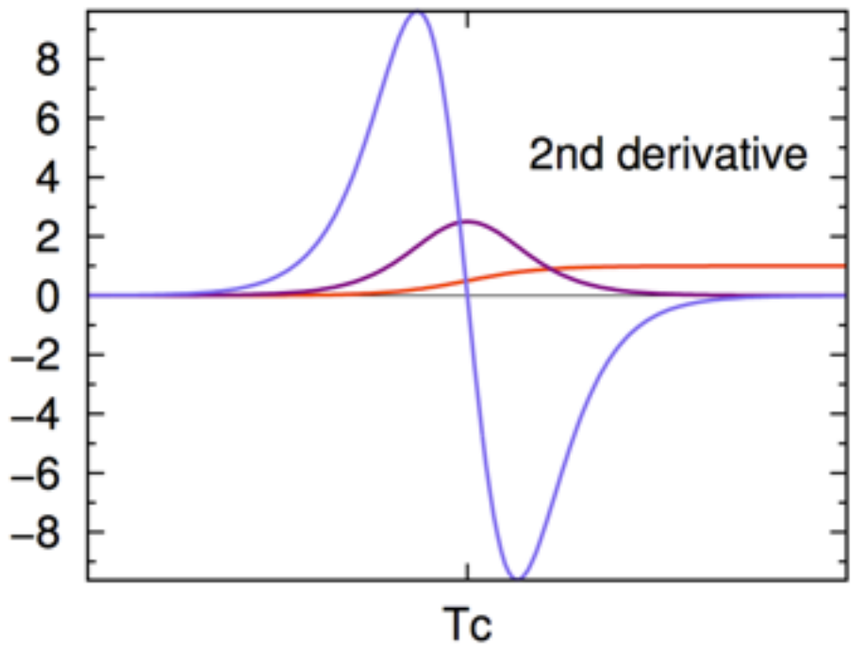

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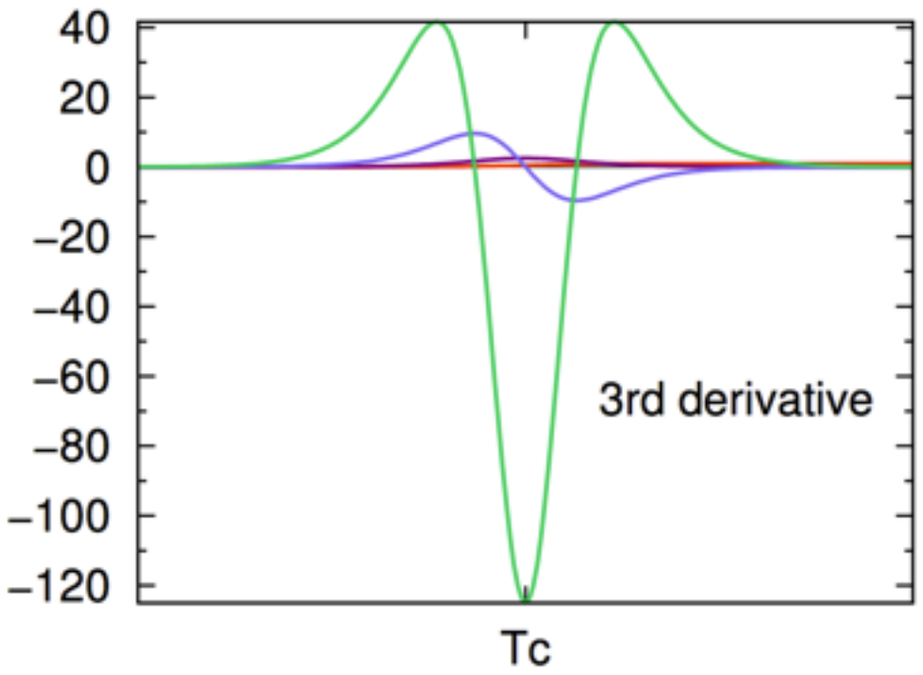

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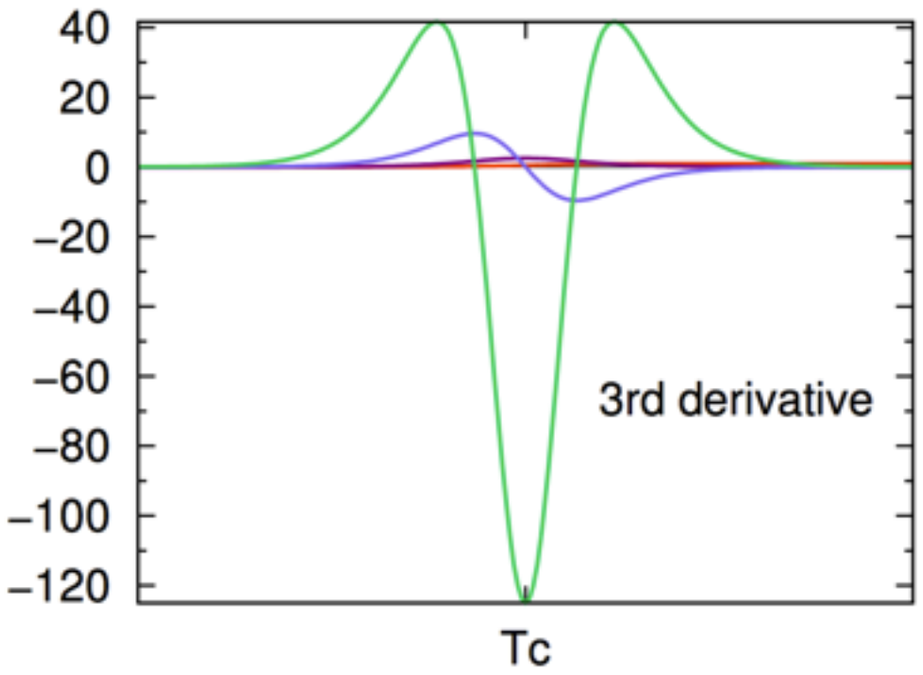


# Fluctuation observables

* **Susceptibilities:**

$$\chi_n = \left. \frac{\partial^n p(T,\mu)/T^4}{\partial(\mu/T)^n} \right|_T$$

strong temperature & density dependence of ratios

* **Fluctuations of conserved charges**

$$\delta Q_X = Q_X - \langle Q_X \rangle \qquad X = Q, B, S, \ldots$$

**mean value:** $\chi_1 \sim \langle Q \rangle$

$$\chi_2 \sim \langle (\delta Q)^2 \rangle$$

$$\chi_3 \sim \langle (\delta Q)^3 \rangle$$

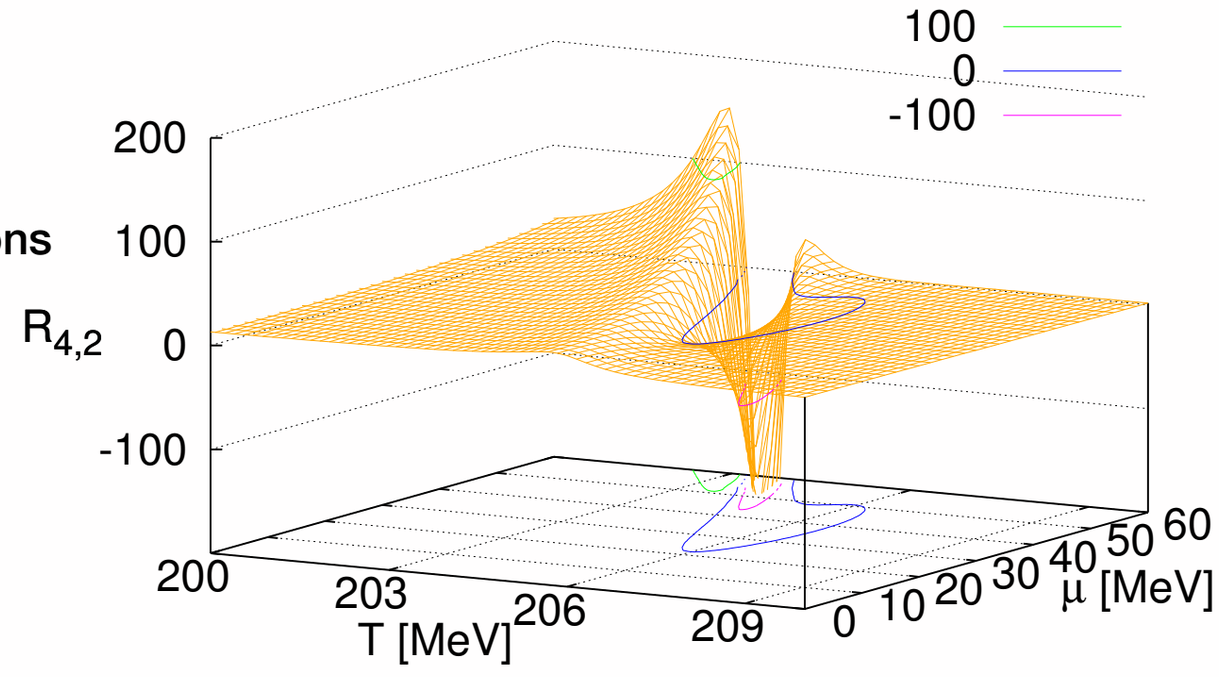
[ BJS, M. Wagner 2012]

* **Measured in event-by-event multiplicity distributions**

**variance:** $\sigma^2 \sim \frac{\chi_2}{\chi_1}$

**skewness:** $S\sigma = \frac{\chi_3}{\chi_2}$

**kurtosis:** $\kappa\sigma^2 = \frac{\chi_4}{\chi_2}$




[ STAR Coll. 2014; PHENIX Coll 1506.07834]

# Infinite volume

**Partition function:**

$$\mathcal{Z}(T,\mu) = \int \underbrace{\mathcal{D}\bar{\psi}\mathcal{D}\psi\cancel{\mathcal{D}\phi}}\, e^{-\int d^4x\, \mathcal{L}(\bar{\psi},\psi,\phi)}$$

replace with (const.) condensate $\sigma$

**Integration of quarks, neglect bosonic fluctuations**

**Grand potential** (Polyakov-)quark-meson model (quark loop)

$$\Omega(T,\mu;\sigma) = \Omega_{\rm vac} + \Omega_{\rm T} + V_{\rm MF}(\sigma) \qquad (+\mathcal{U}_{\rm Poly}(\Phi)\ )$$

**Vacuum term: regularize e.g. with sharp O(3)-momentum cutoff**

$$\Omega_{\rm vac}(\Lambda) = -4\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+m_q^2}$$

for each cutoff: adjust model parameters like $f_\pi, m_\sigma, m_\pi$

standard MFA: $\Lambda = 0$

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[A Juricic, BJS to be published]

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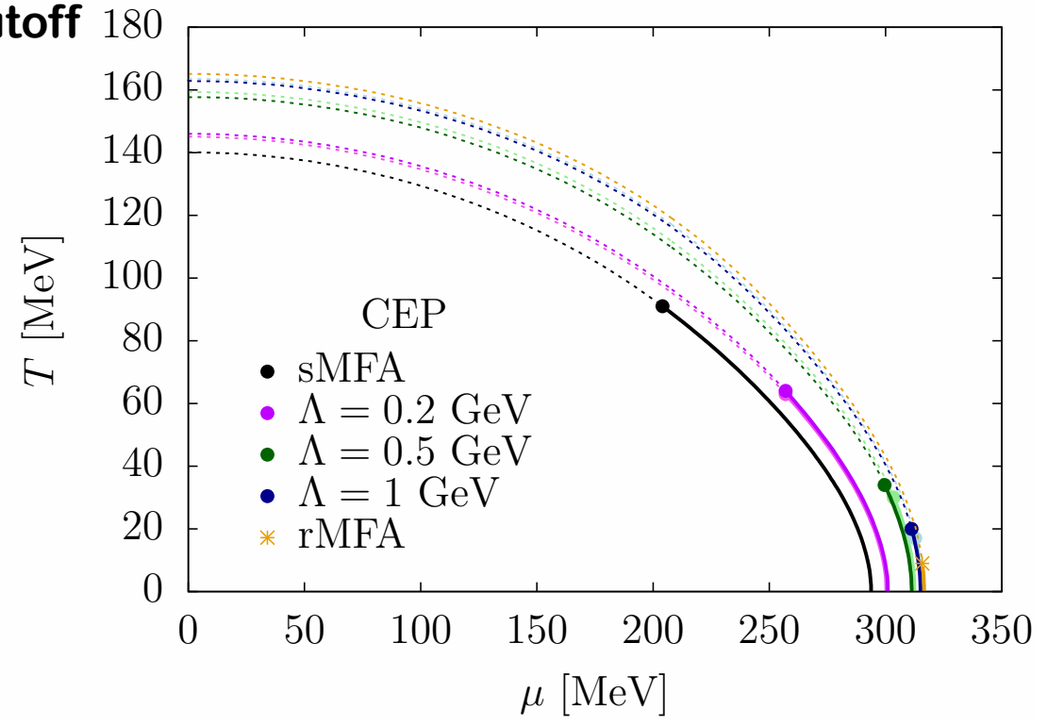
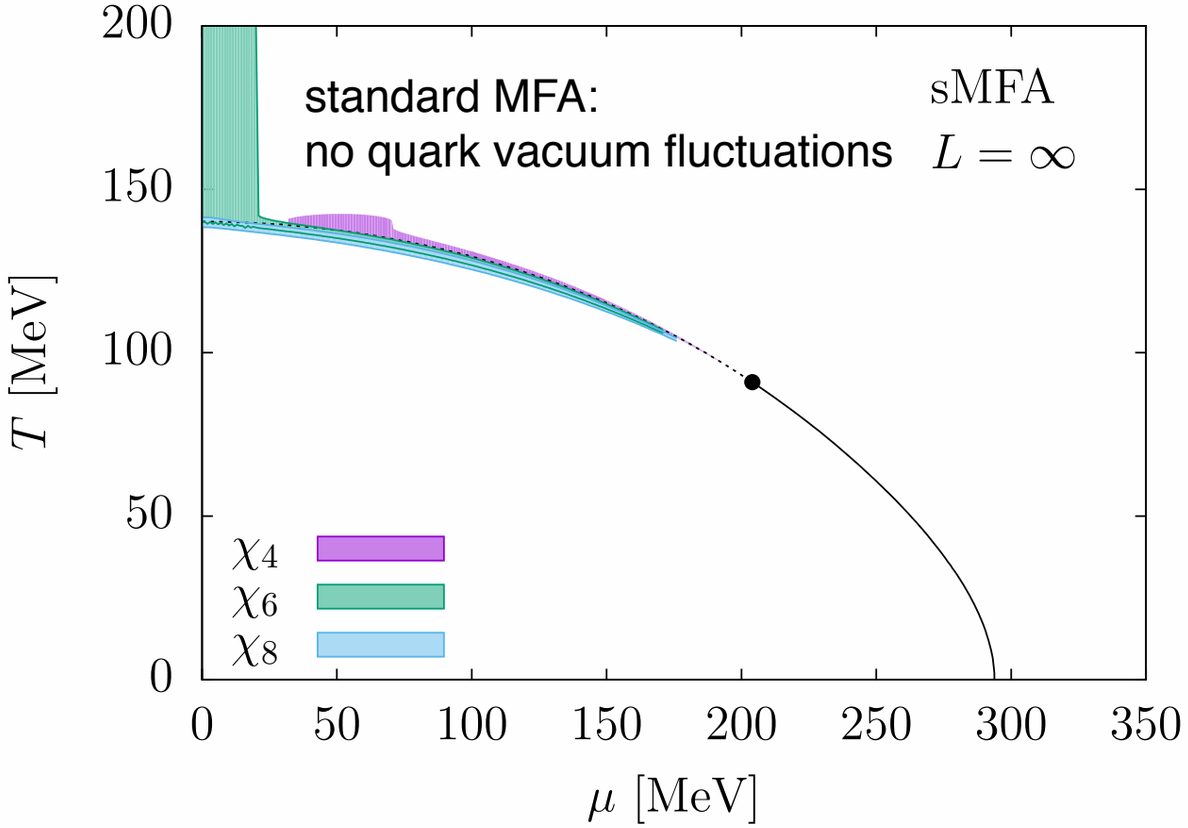
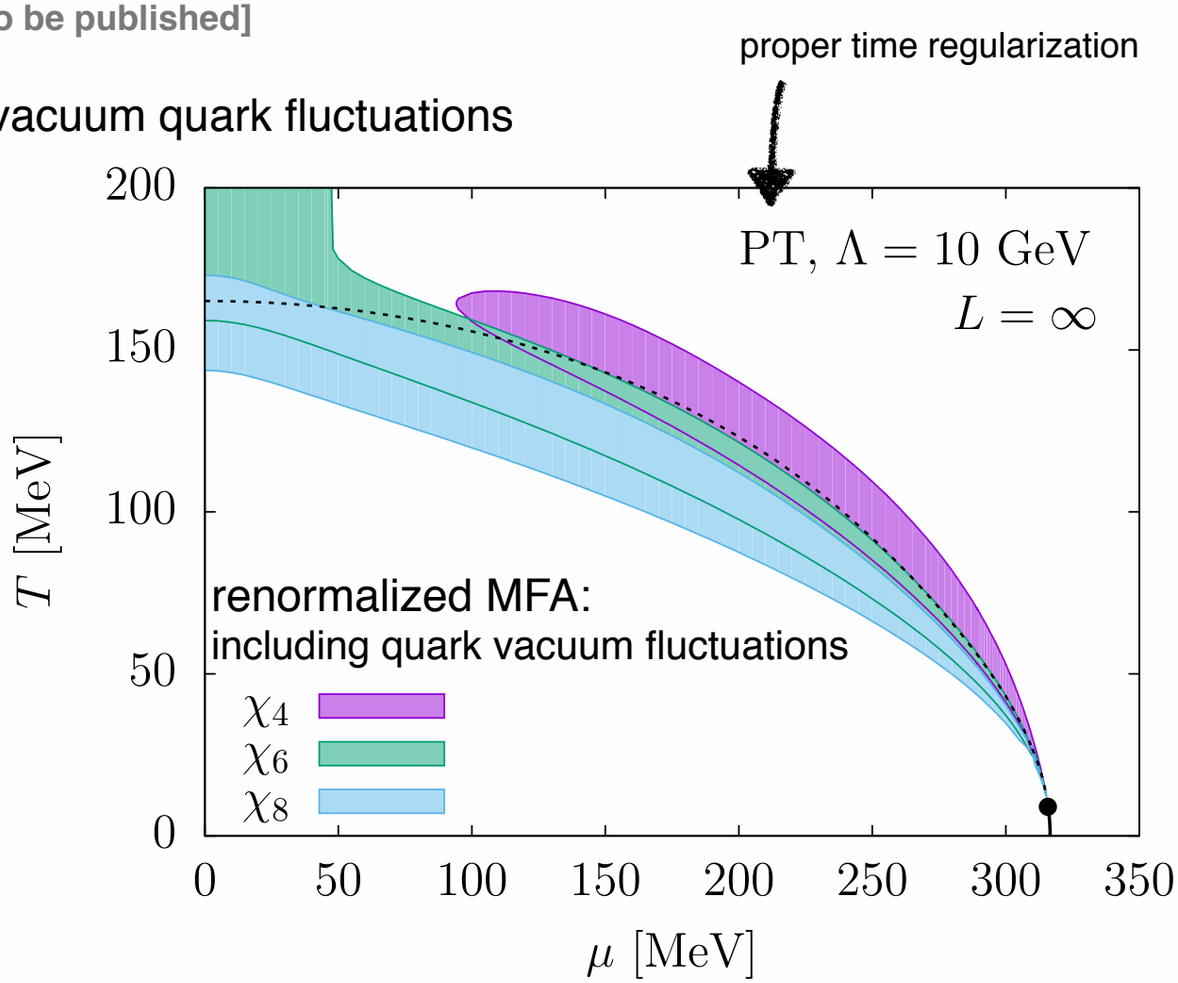


# Higher cumulants

[A Juricic, BJS to be published]

infinite volume: influence of vacuum quark fluctuations



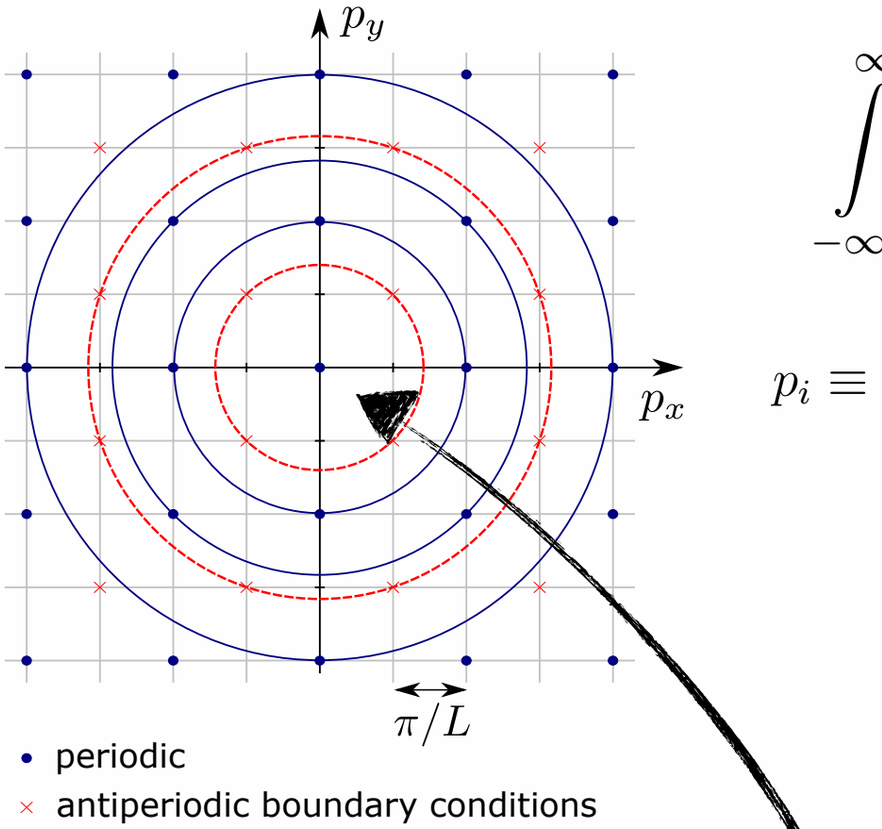




findings:

(quark) fluctuations pushes CEP to smaller T and bigger μ

Fluctuations wash out phase transition ➜ broader negativer regions

# Finite volume




$$\int_{-\infty}^{\infty} \frac{dp_a}{2\pi} \ldots \to \frac{1}{L} \sum_{n_a}$$

$$p_i \equiv \begin{cases} 2\pi T n_i \\ 2\pi T (n_i + \frac{1}{2}) + i\mu \end{cases}$$

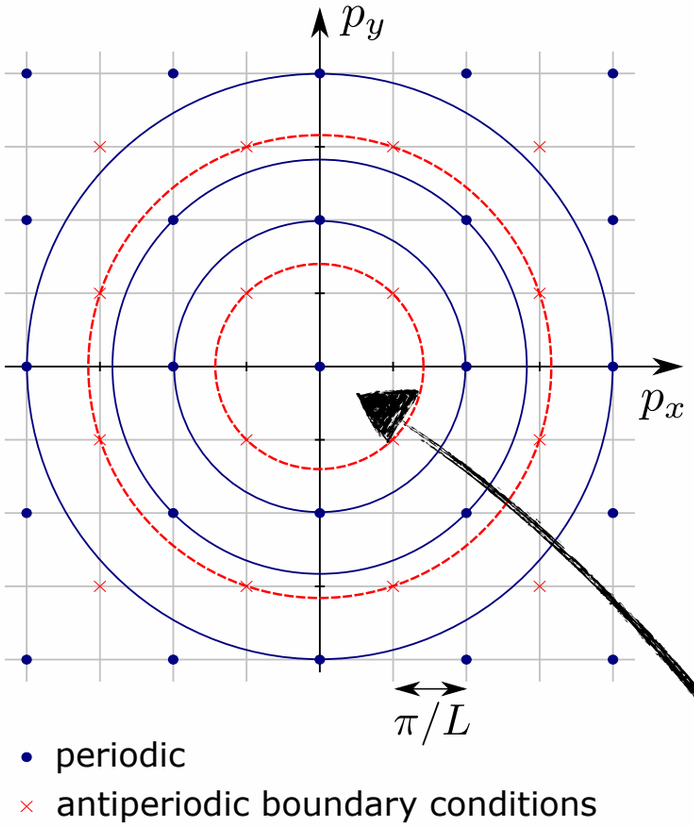
$$T \leftrightarrow 1/L$$

PBC: periodic BC including zero mode

PBC*: PBC without zero mode

ABC: antiperiodic BC

# Finite volume

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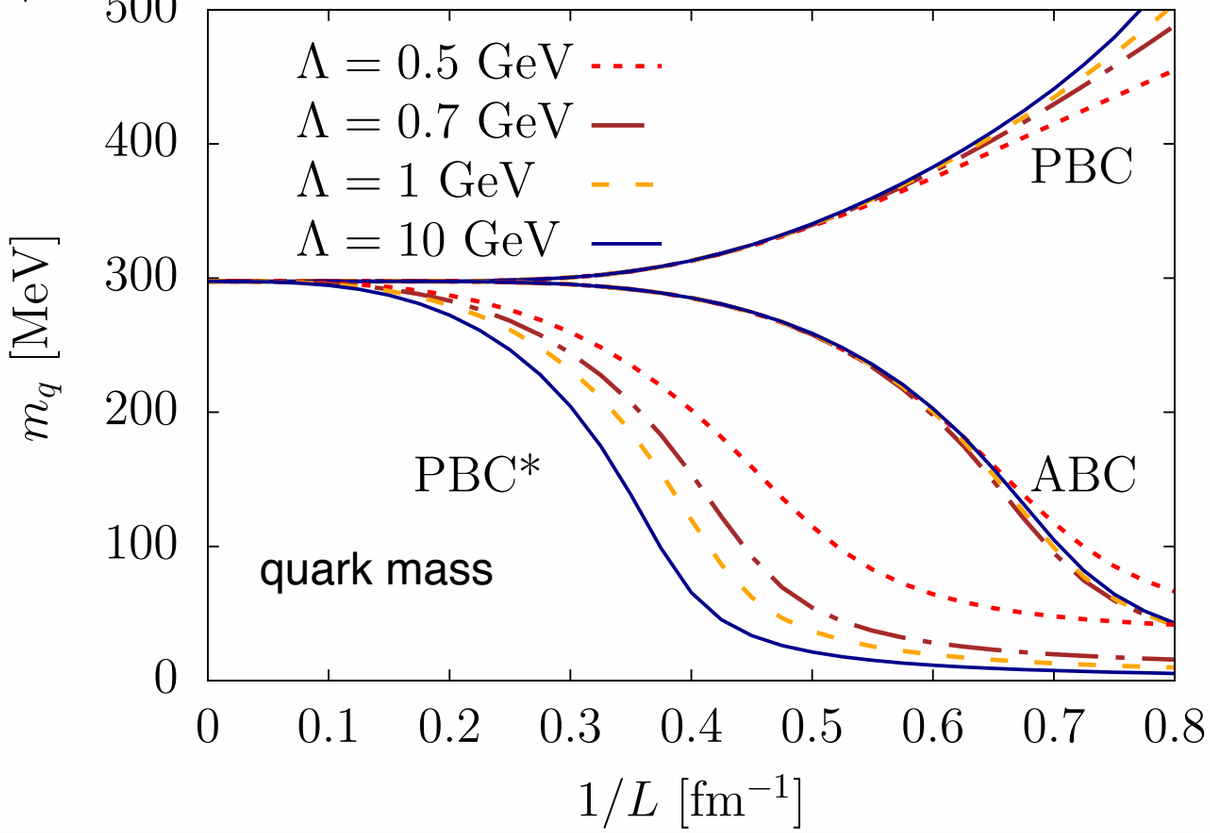
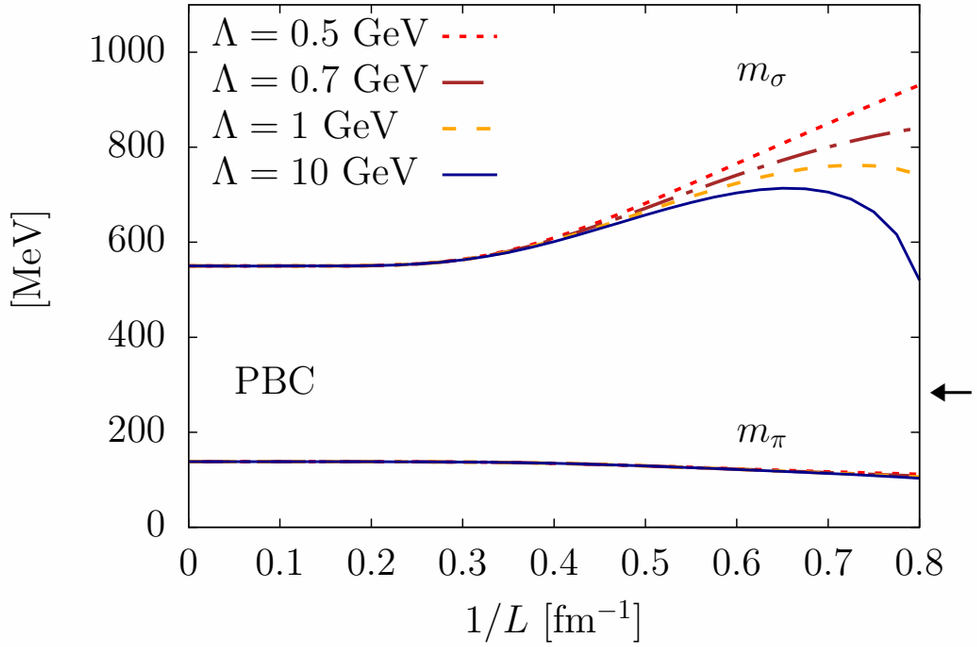


# Vacuum meson masses

[A Juricic, BJS in preparation]

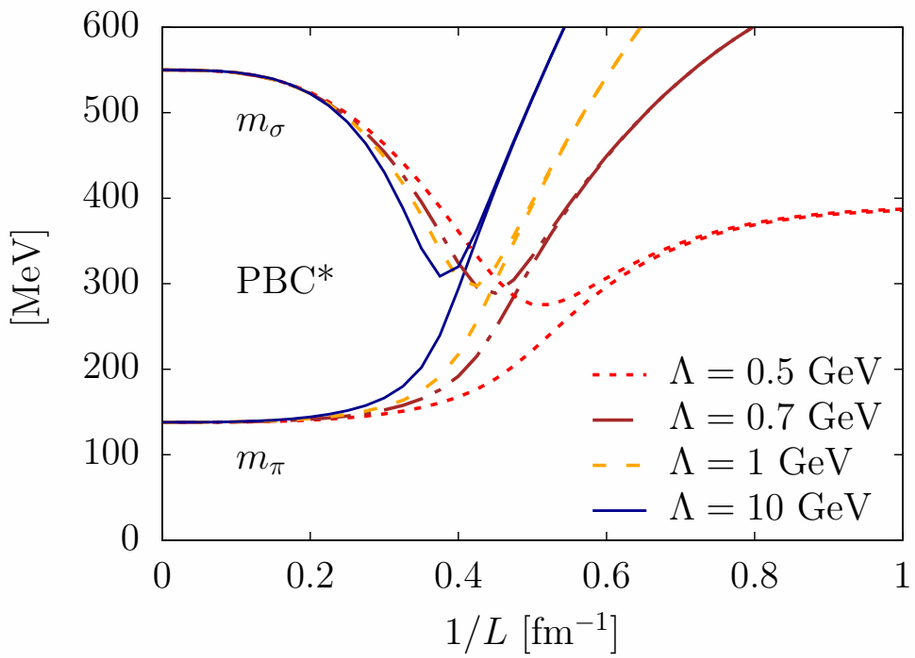


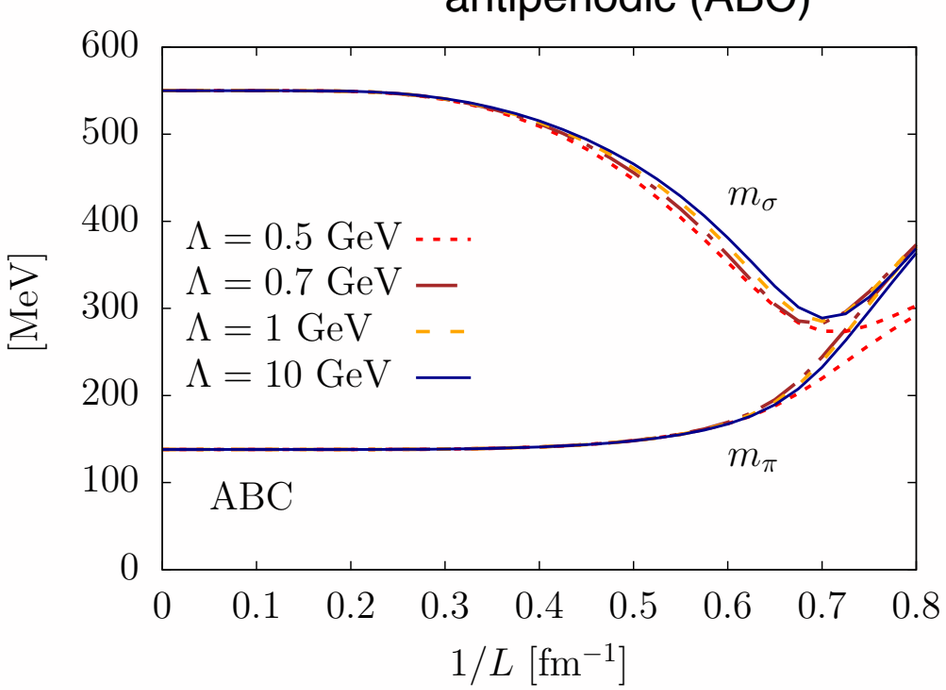

curvature masses (not pole masses)

← periodic including zero-mode (PBC)

periodic without zero-mode (PBC*) ↙

antiperiodic (ABC)



# Thermodynamics on a torus

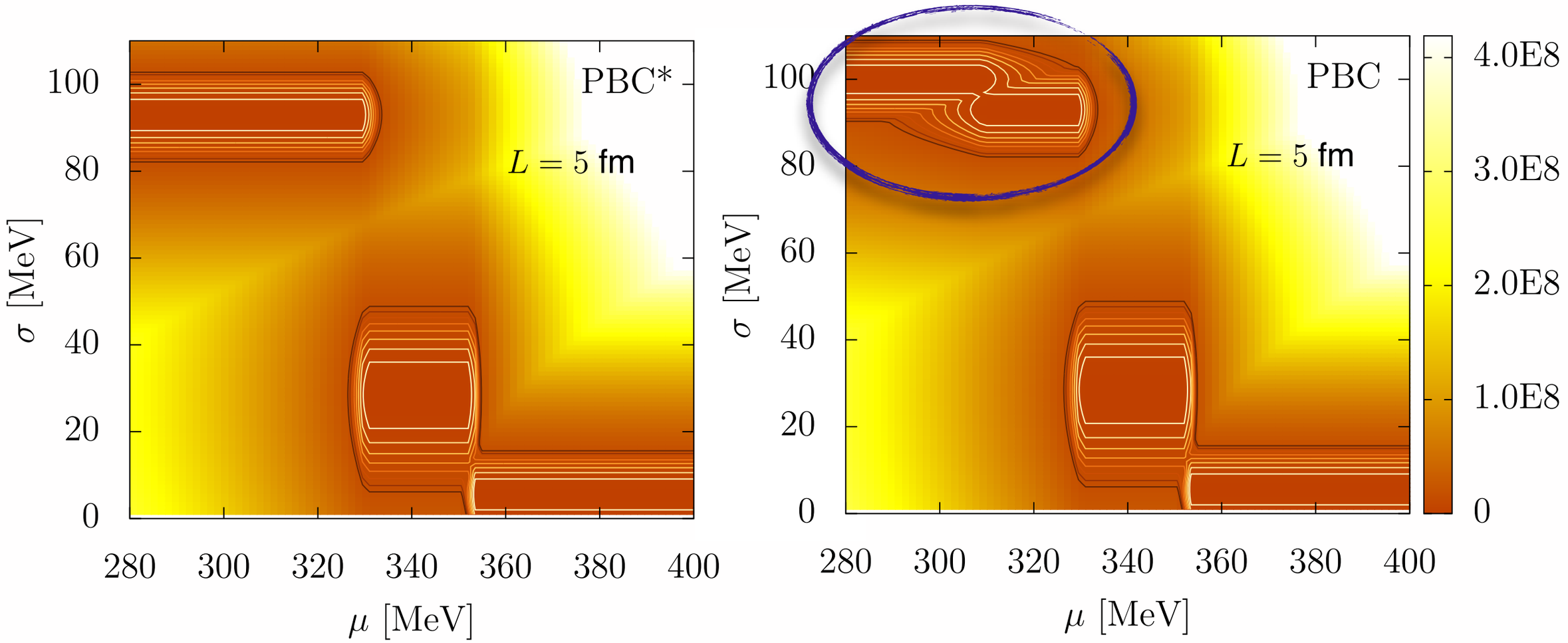
grand potential T=0 & μ>0:

[A Juricic, BJS in preparation]

$$U^{\text{therm}} = 2N_c N_f \frac{1}{L^3} \sum_{\vec{n}\in\mathbb{Z}^3} (\mu - E_{q,\ell})\Theta(\mu - E_{q,\ell})$$

for each mode: discontinuous jumps in potential

contour plot of potential



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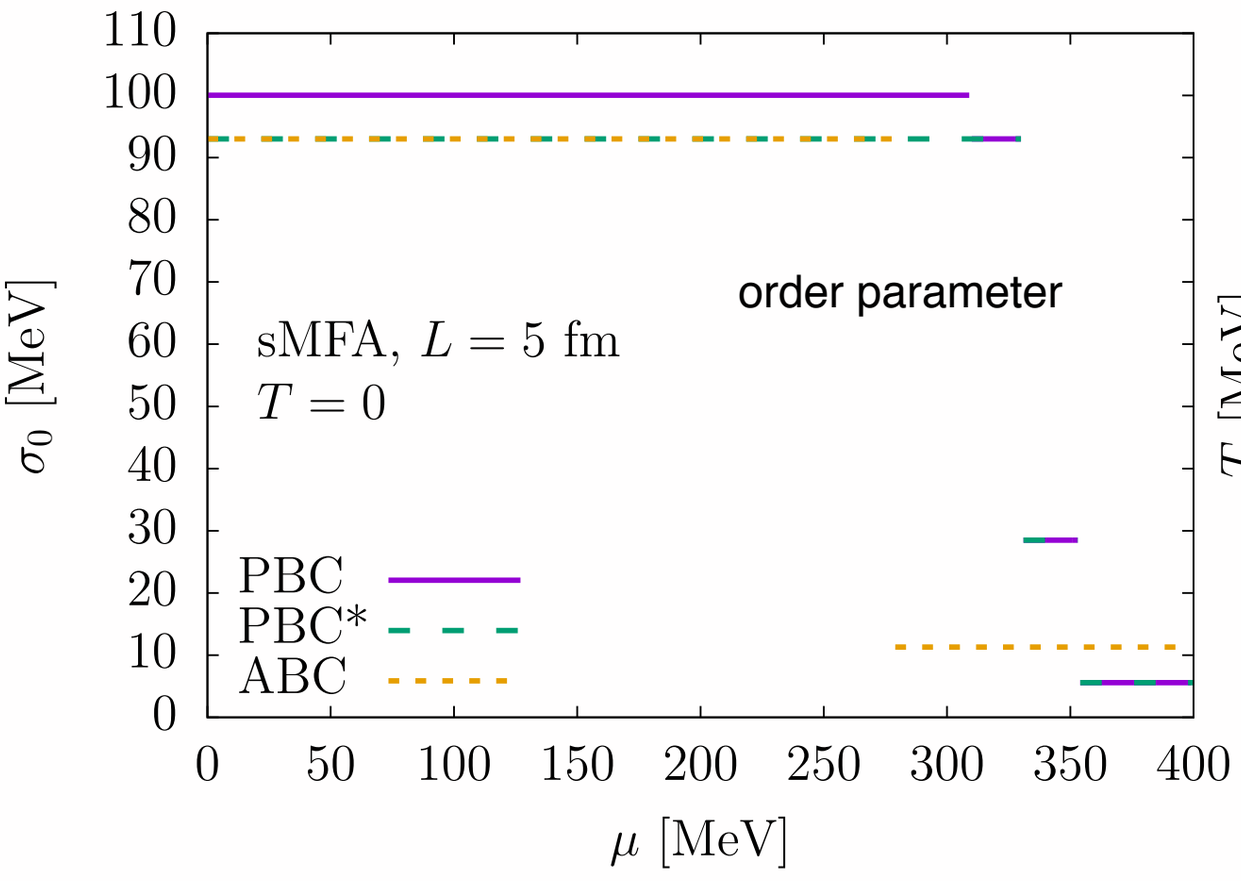
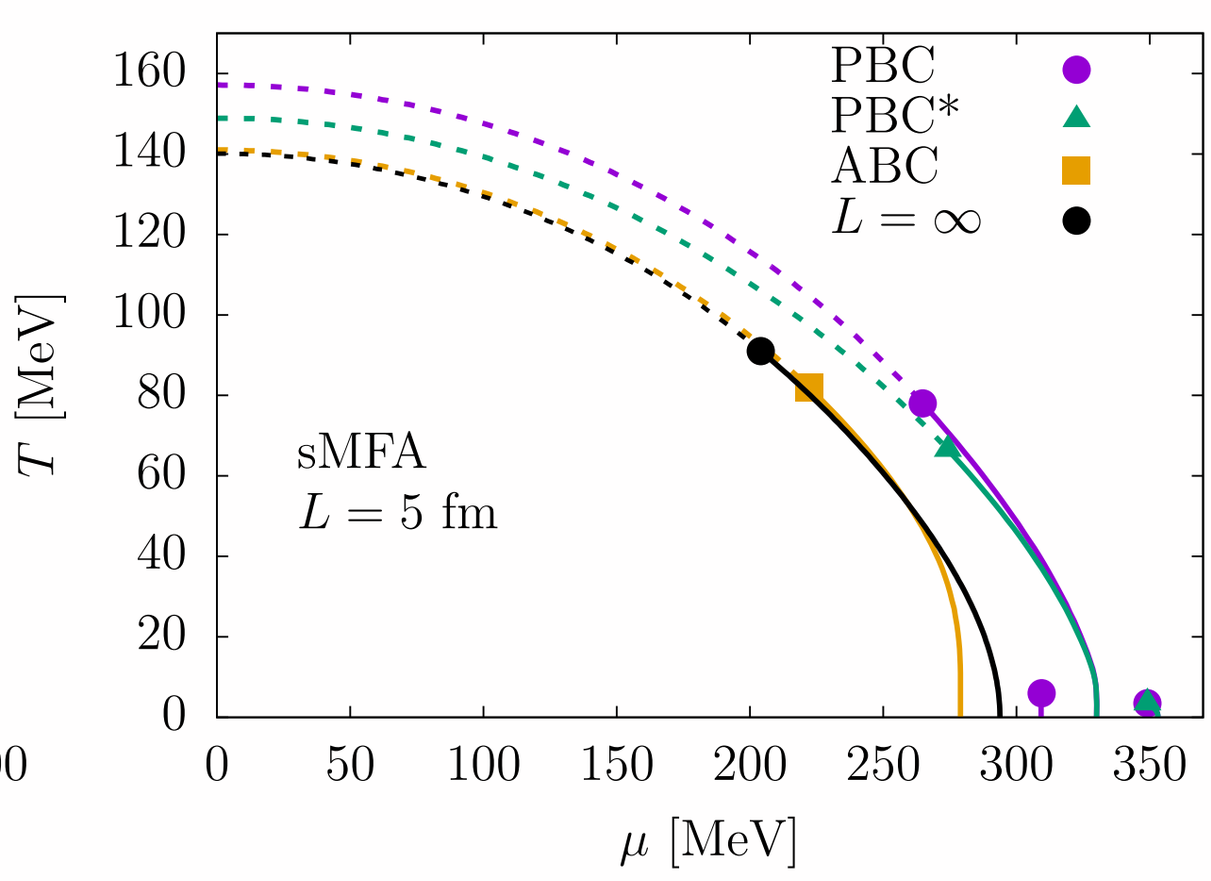
discontinuous jumps washed out at T>0




phase diagram without vacuum fluctuations



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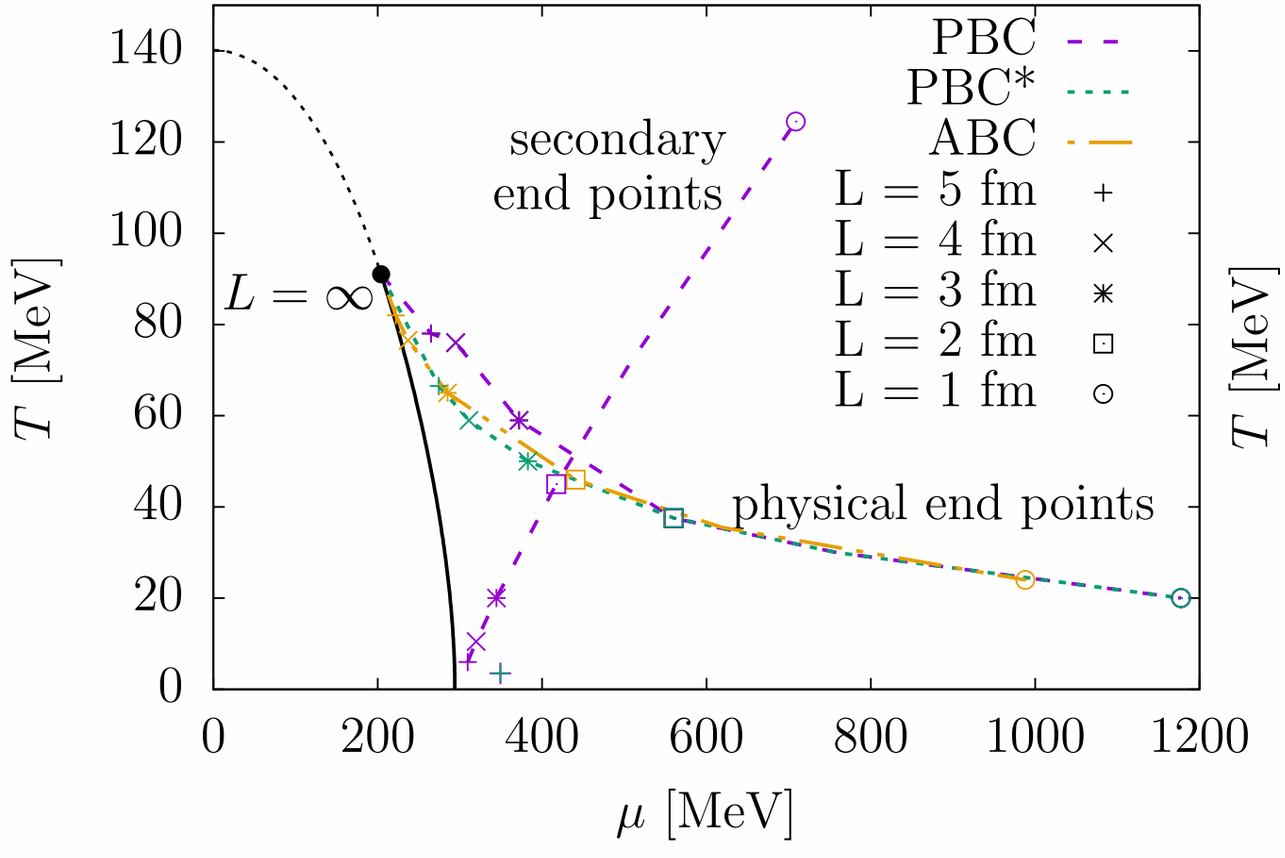
movement of the CEP's

standard MFA (no vacuum fluctuations)

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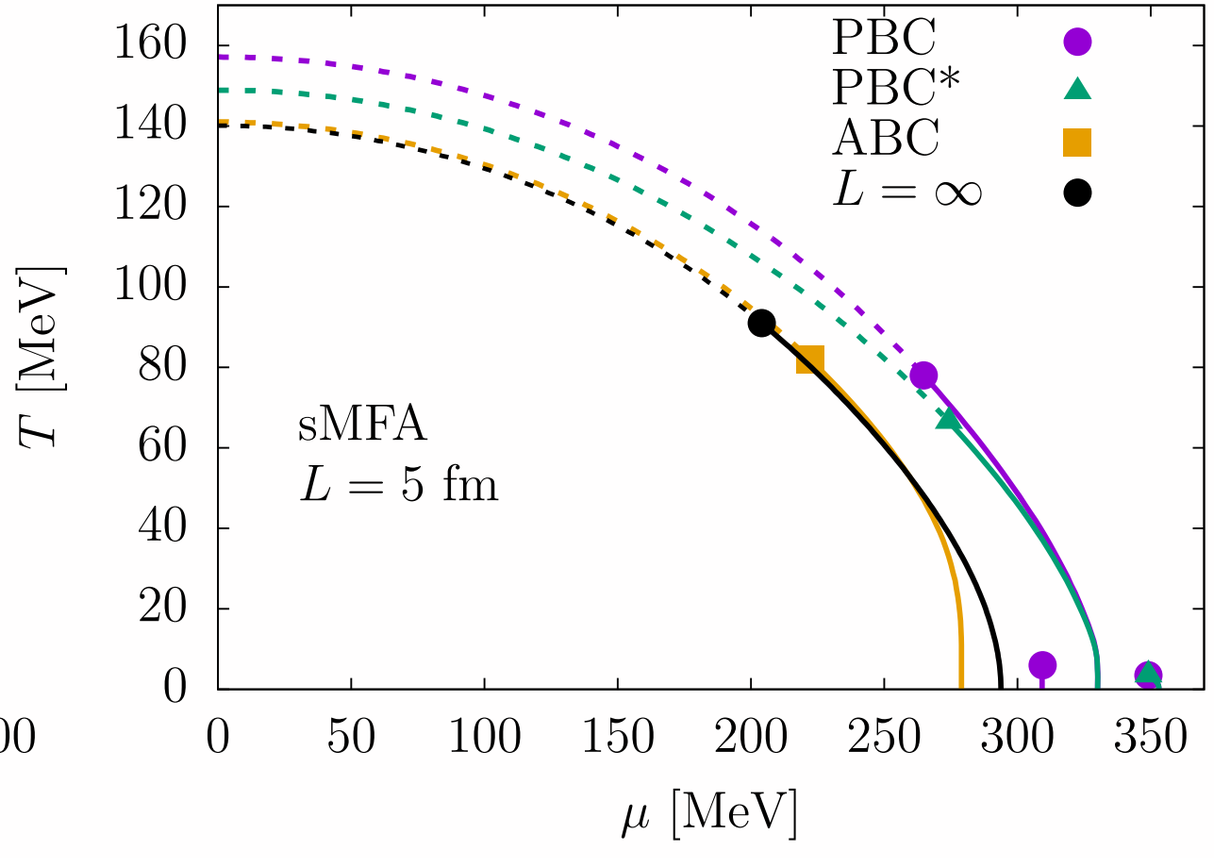
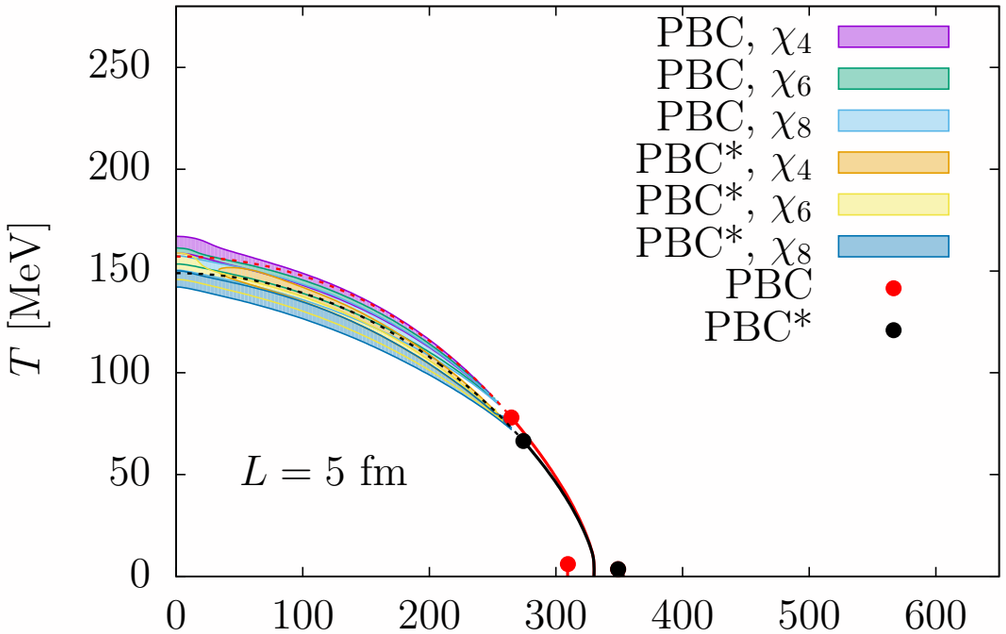
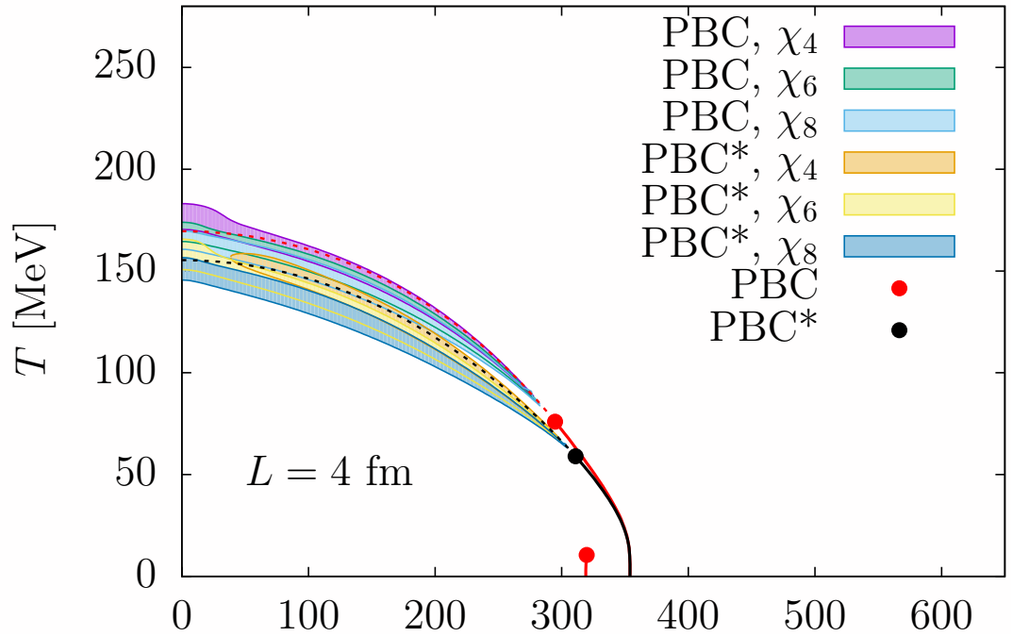


# Cumulants

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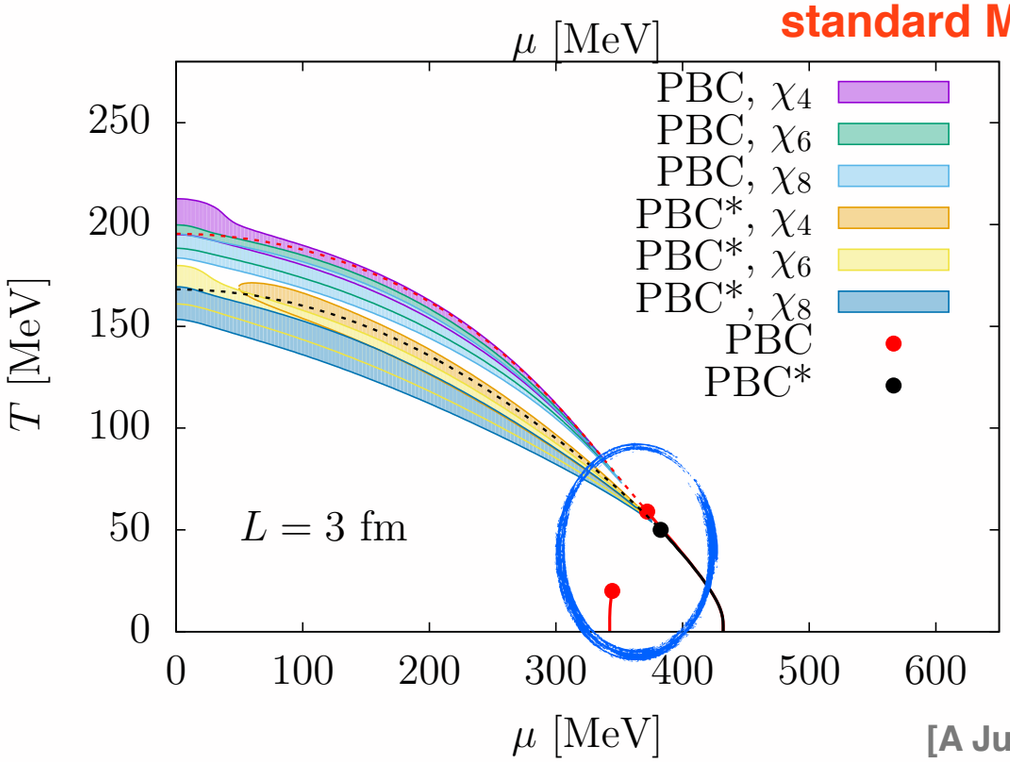


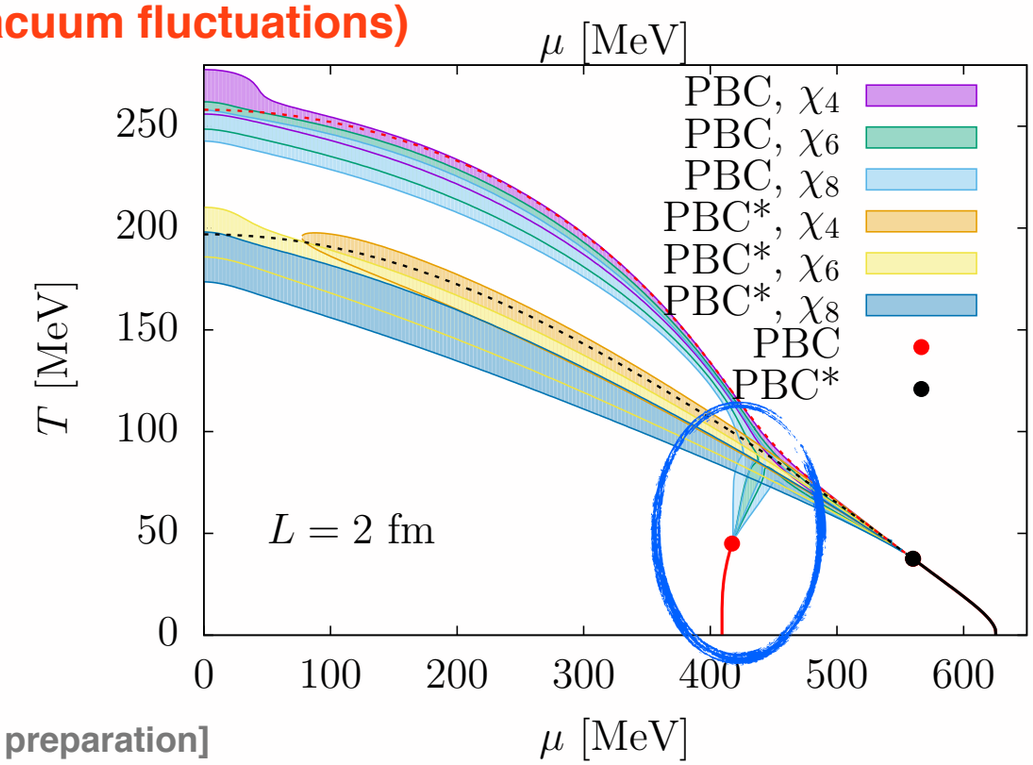


[A Juricic, BJS in preparation]

# Vacuum fluctuations

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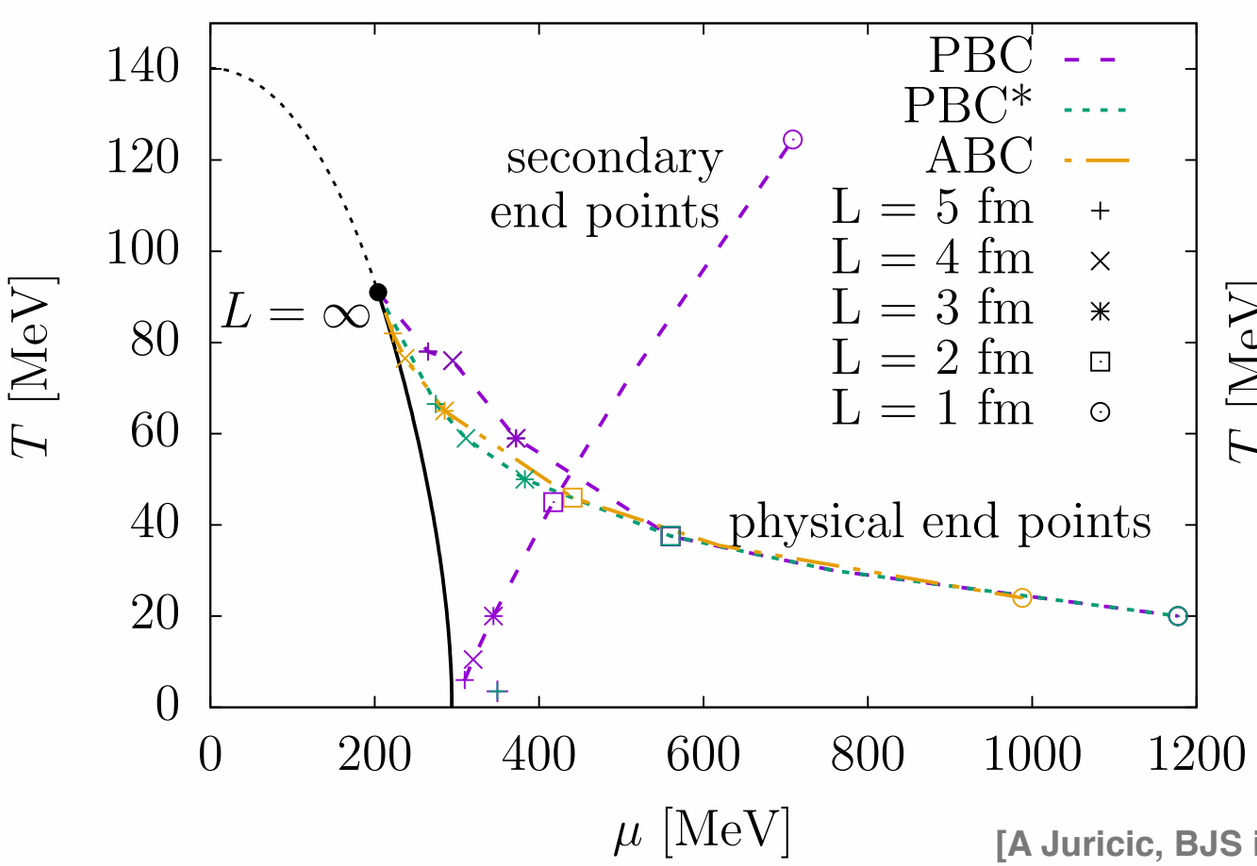
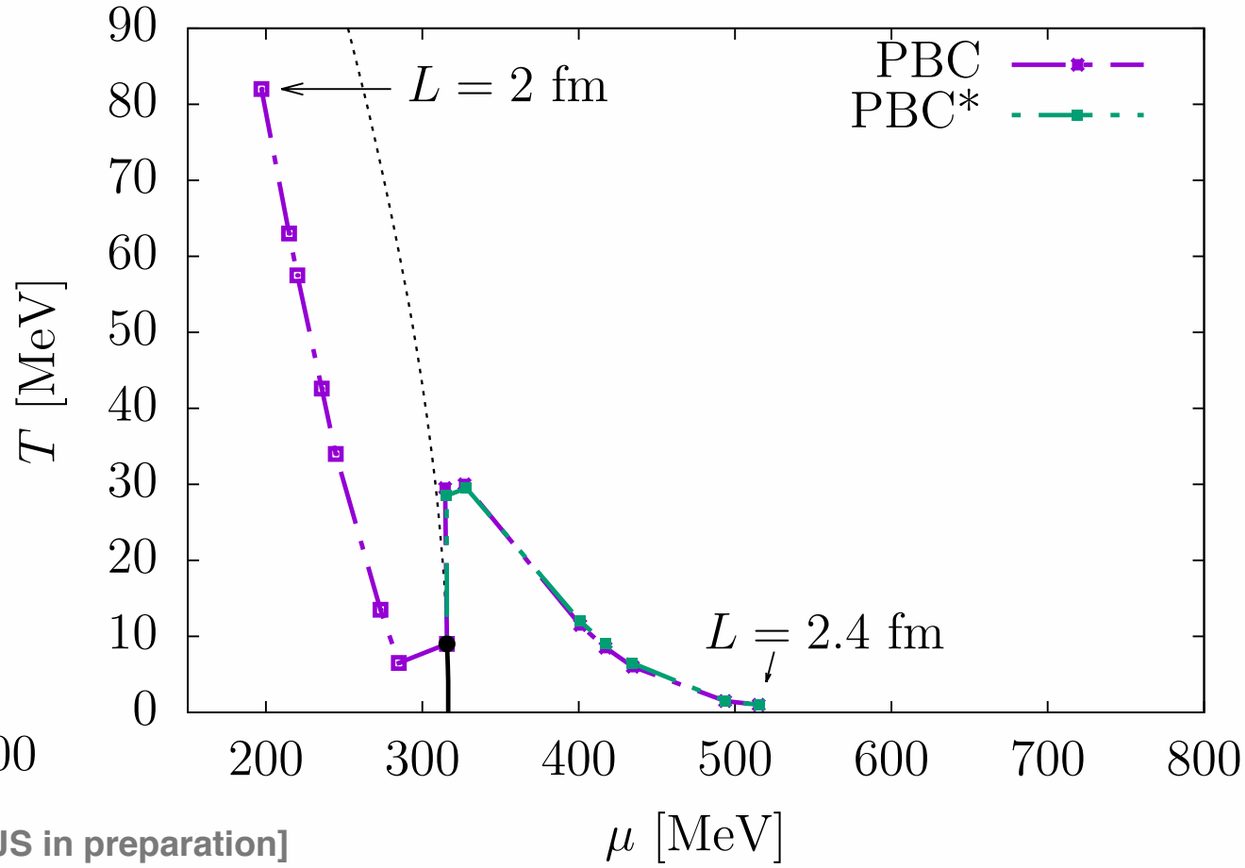



**renormalizes MFA (with vacuum fluctuations)**

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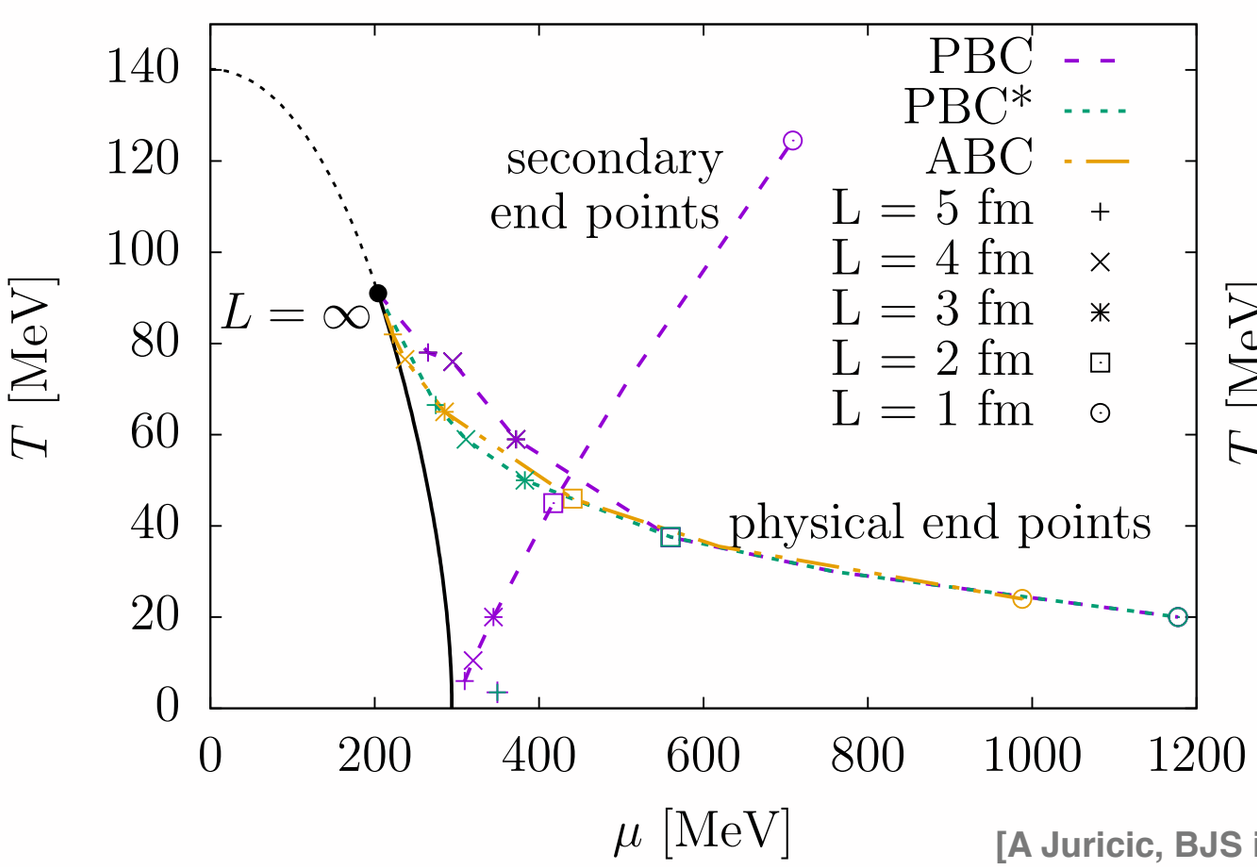


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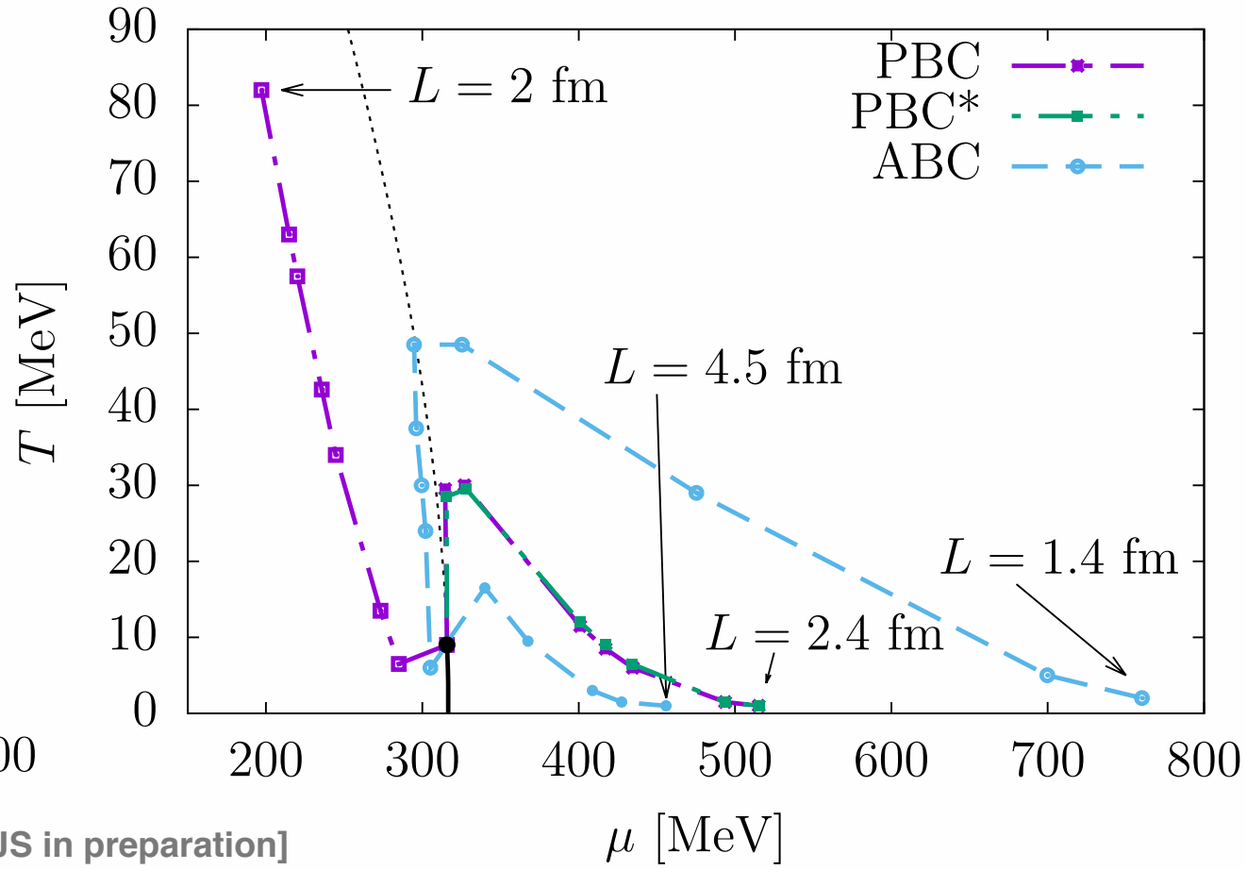


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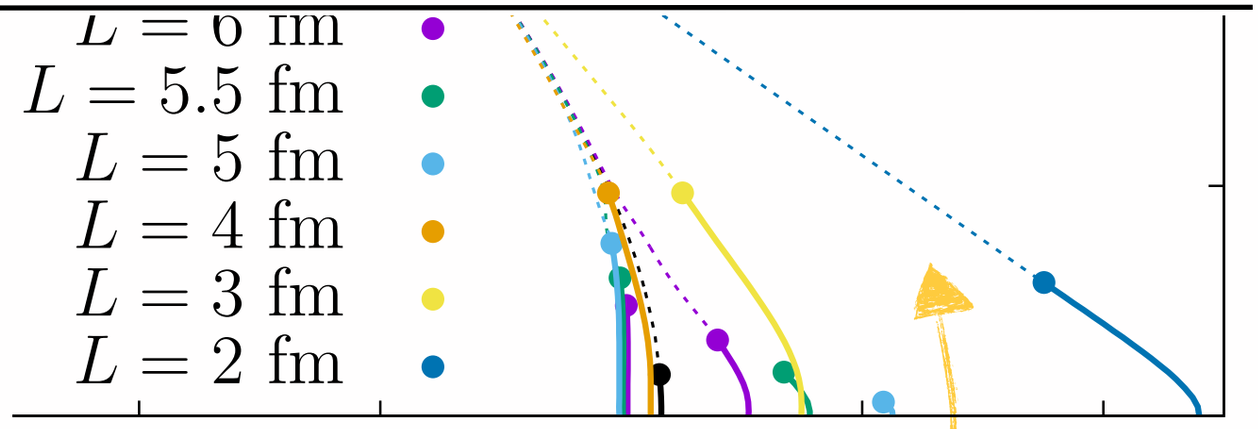
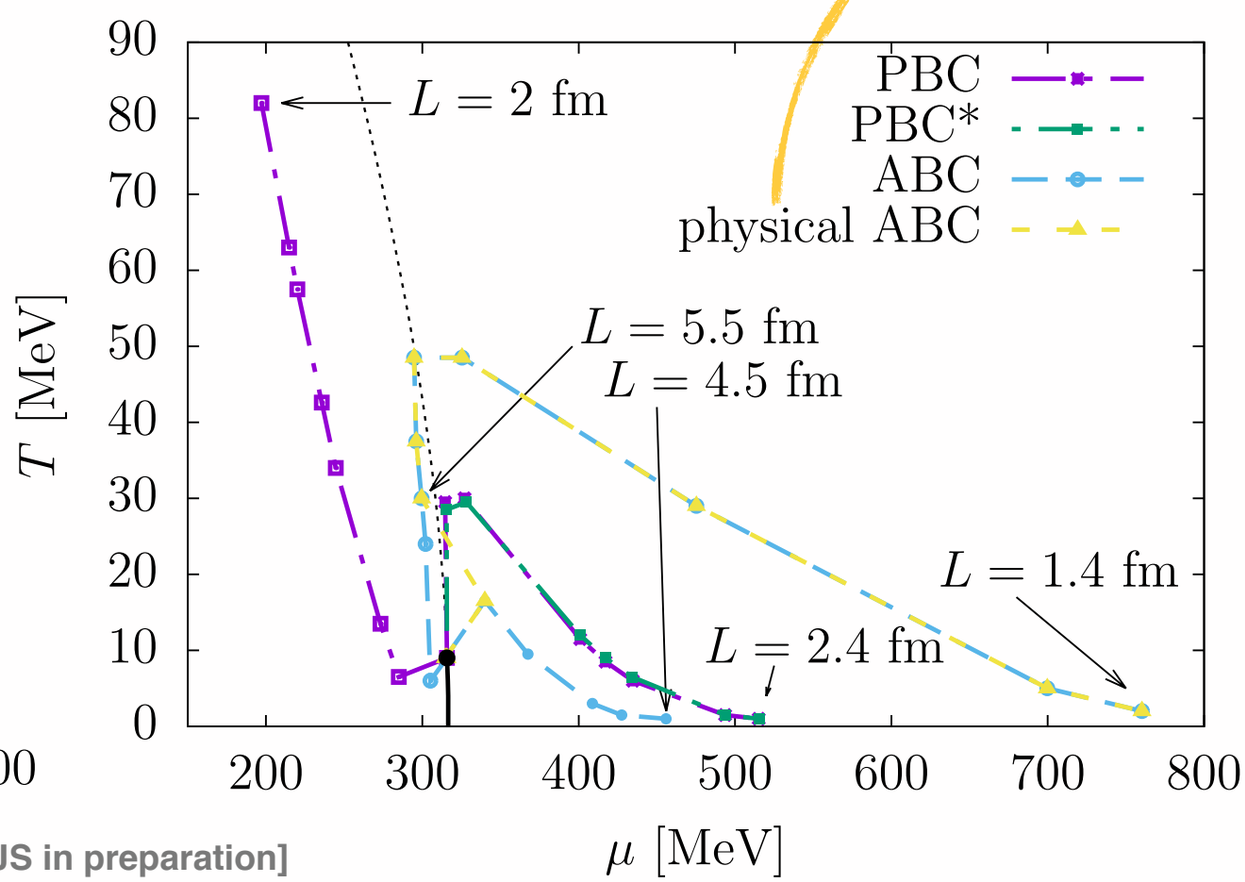



movement of the CEP's

standard MFA (no vacuum fluctuations)



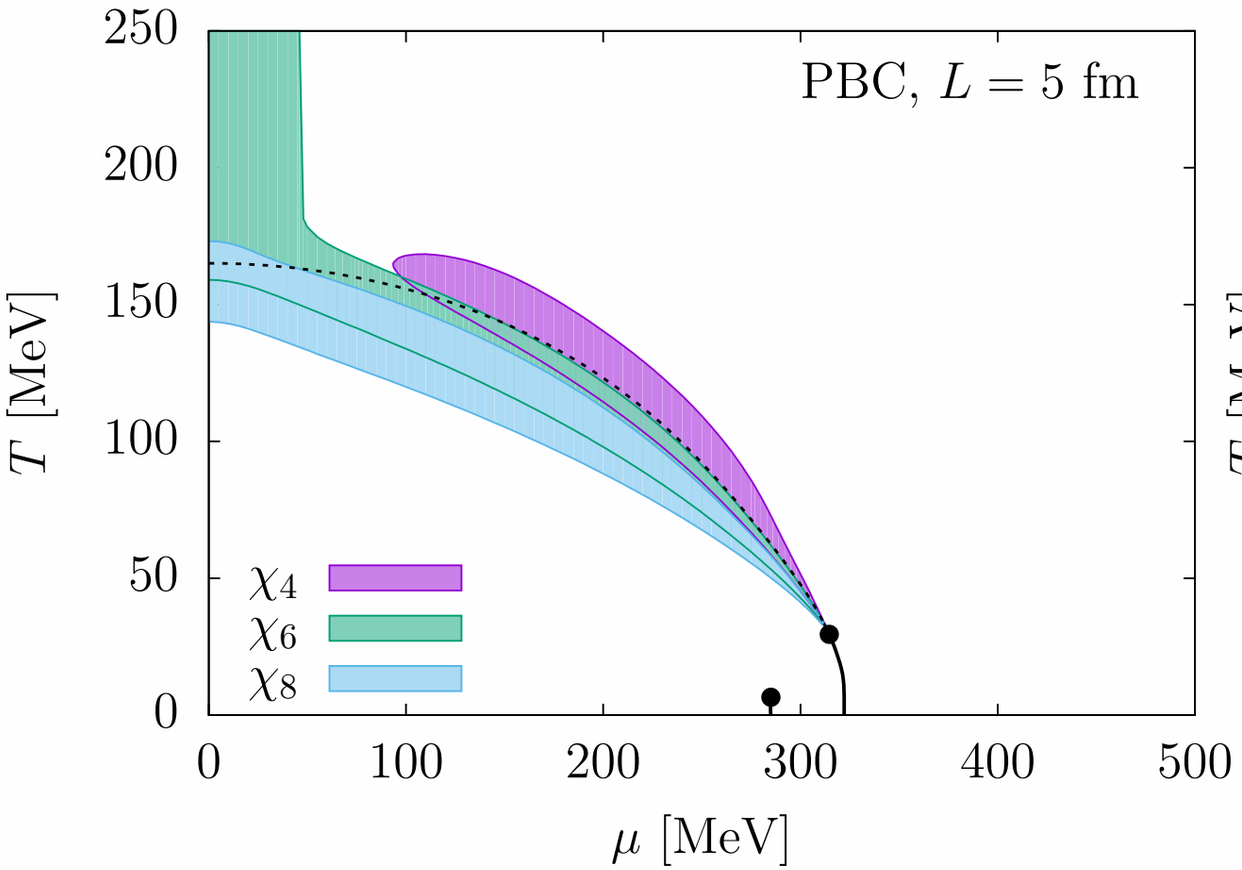

**renormalizes MFA (with vacuum fluctuations)**

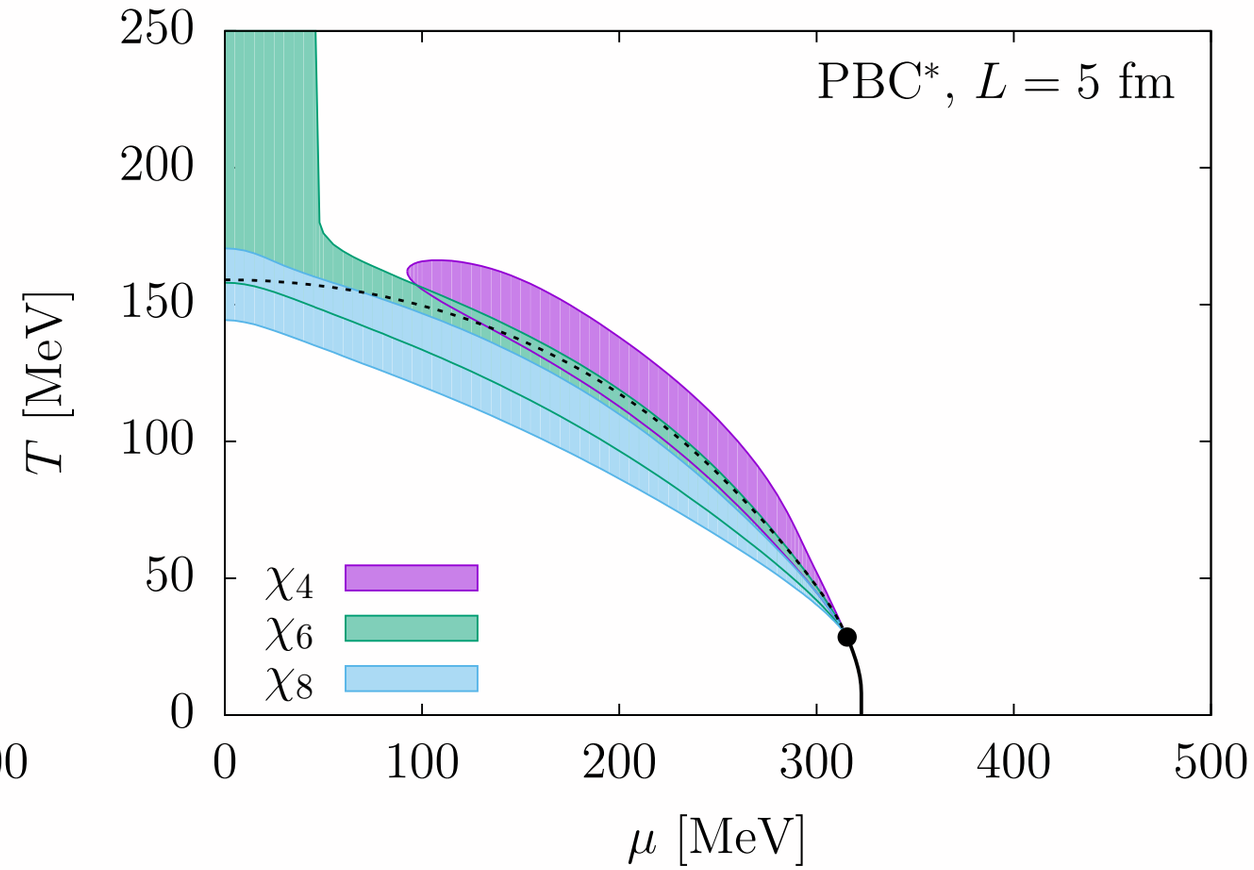



[A Juricic, BJS in preparation]

# Vacuum fluctuations

**renormalizes MFA (with vacuum fluctuations)**

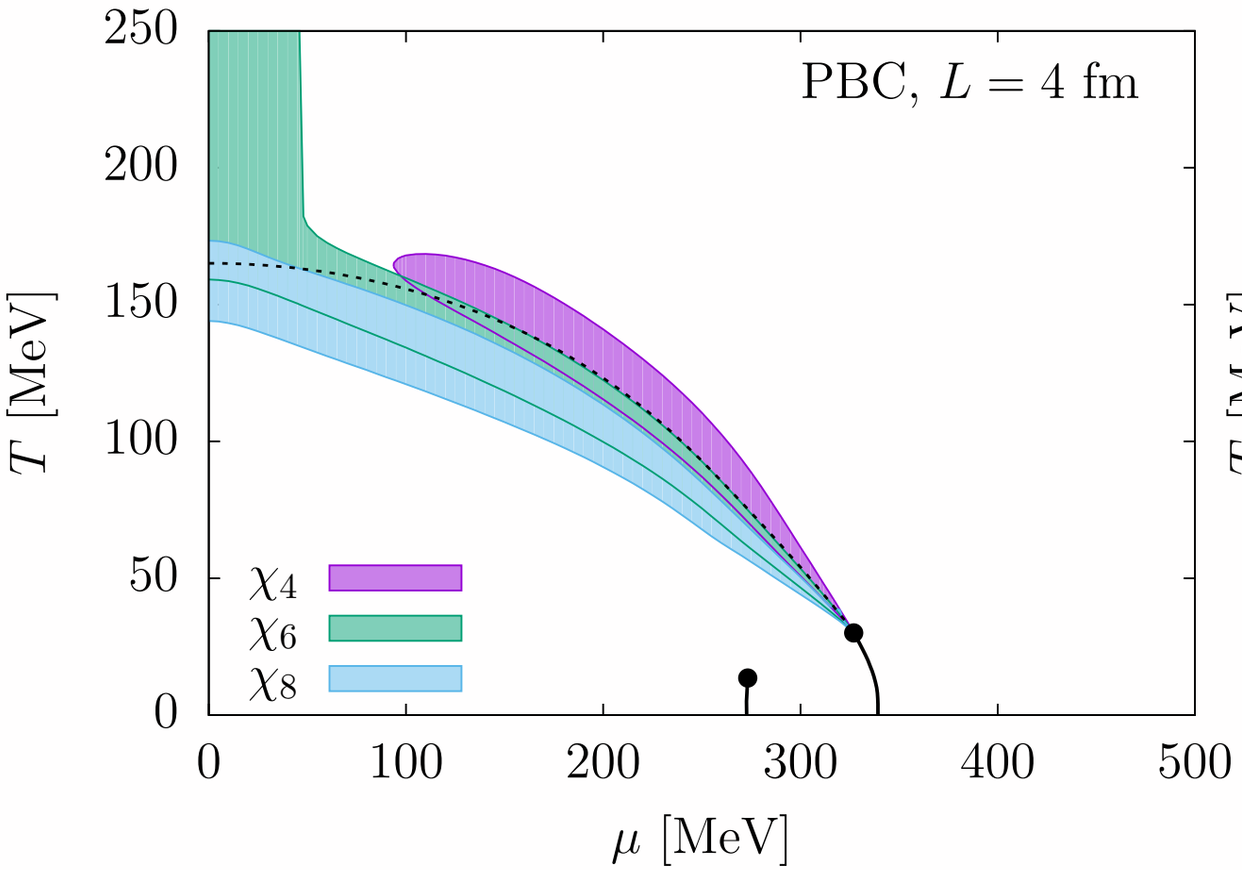


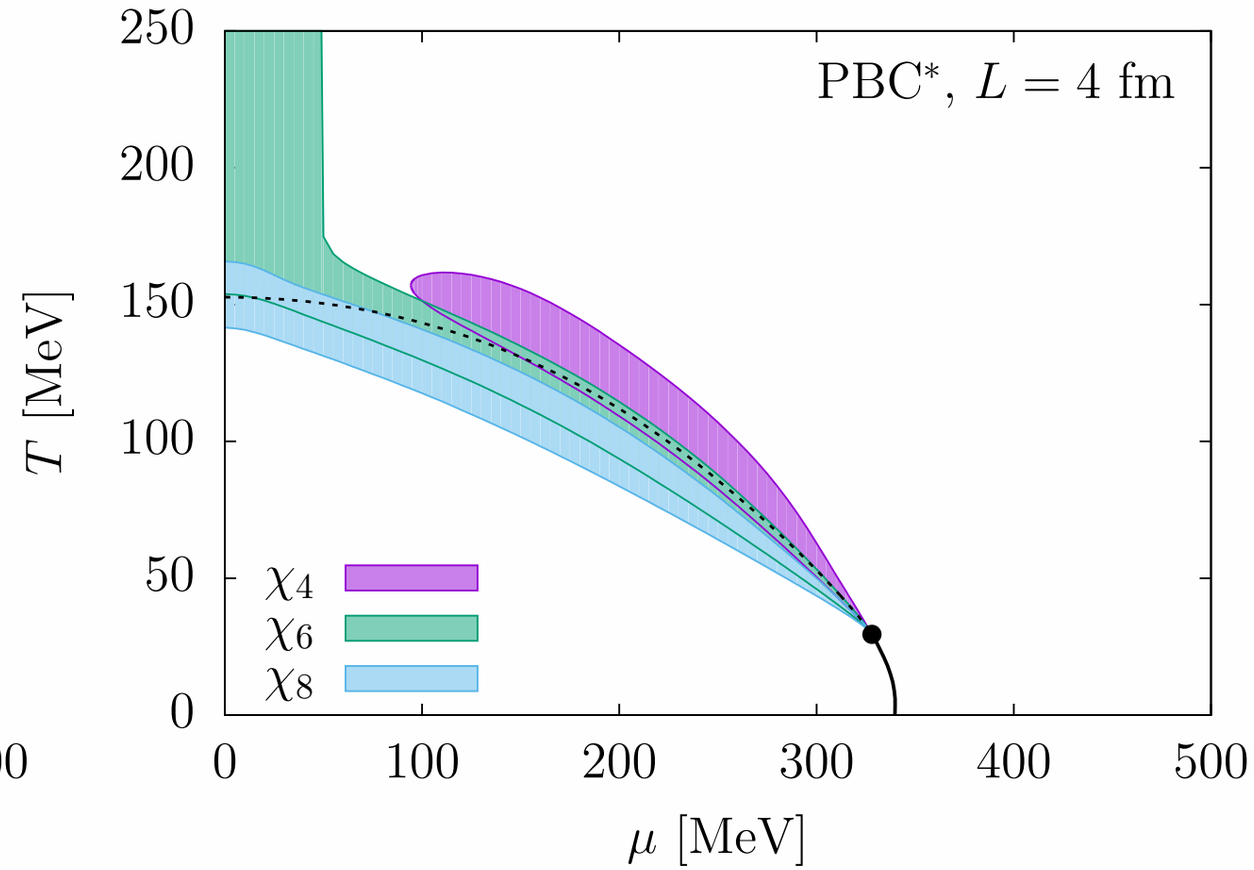




[A Juricic, BJS in preparation]

# Vacuum fluctuations

**renormalizes MFA (with vacuum fluctuations)**

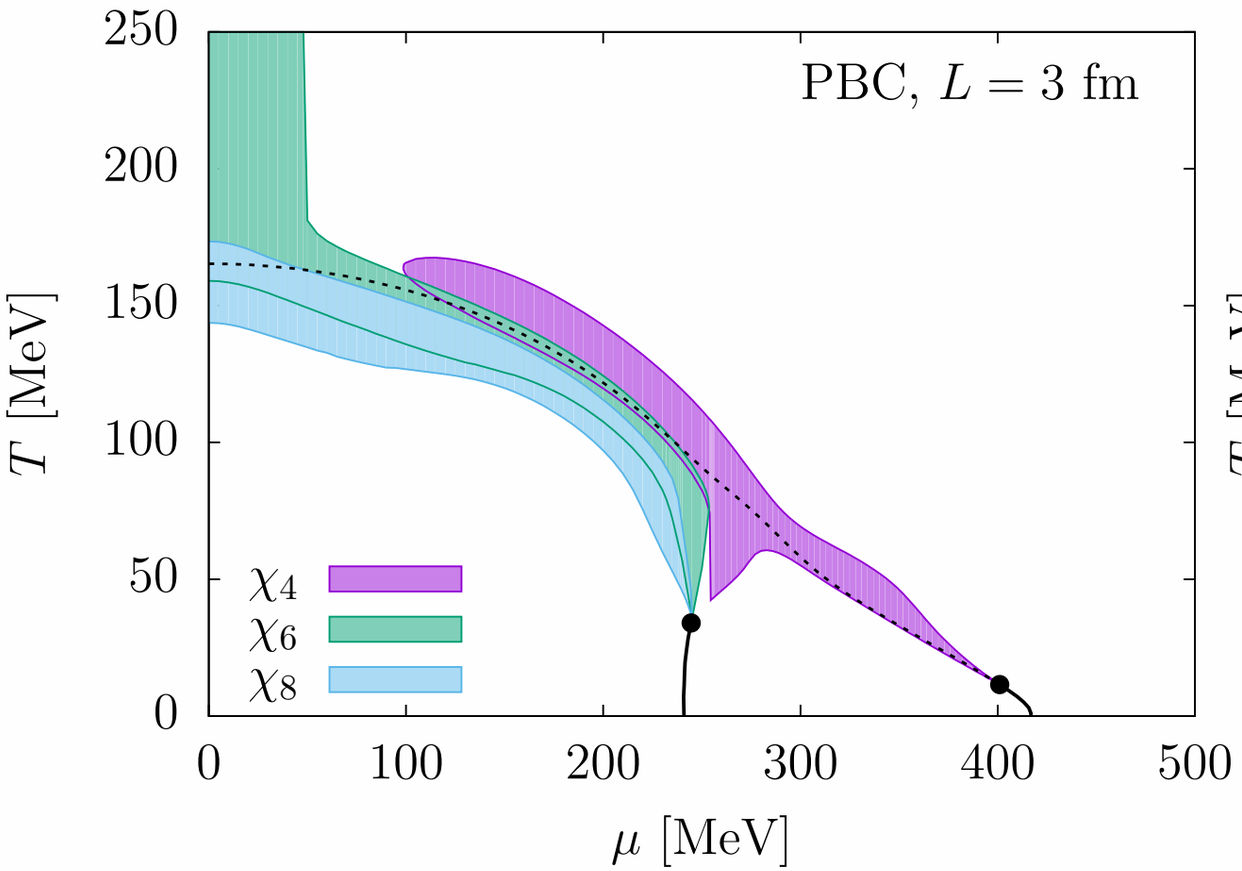


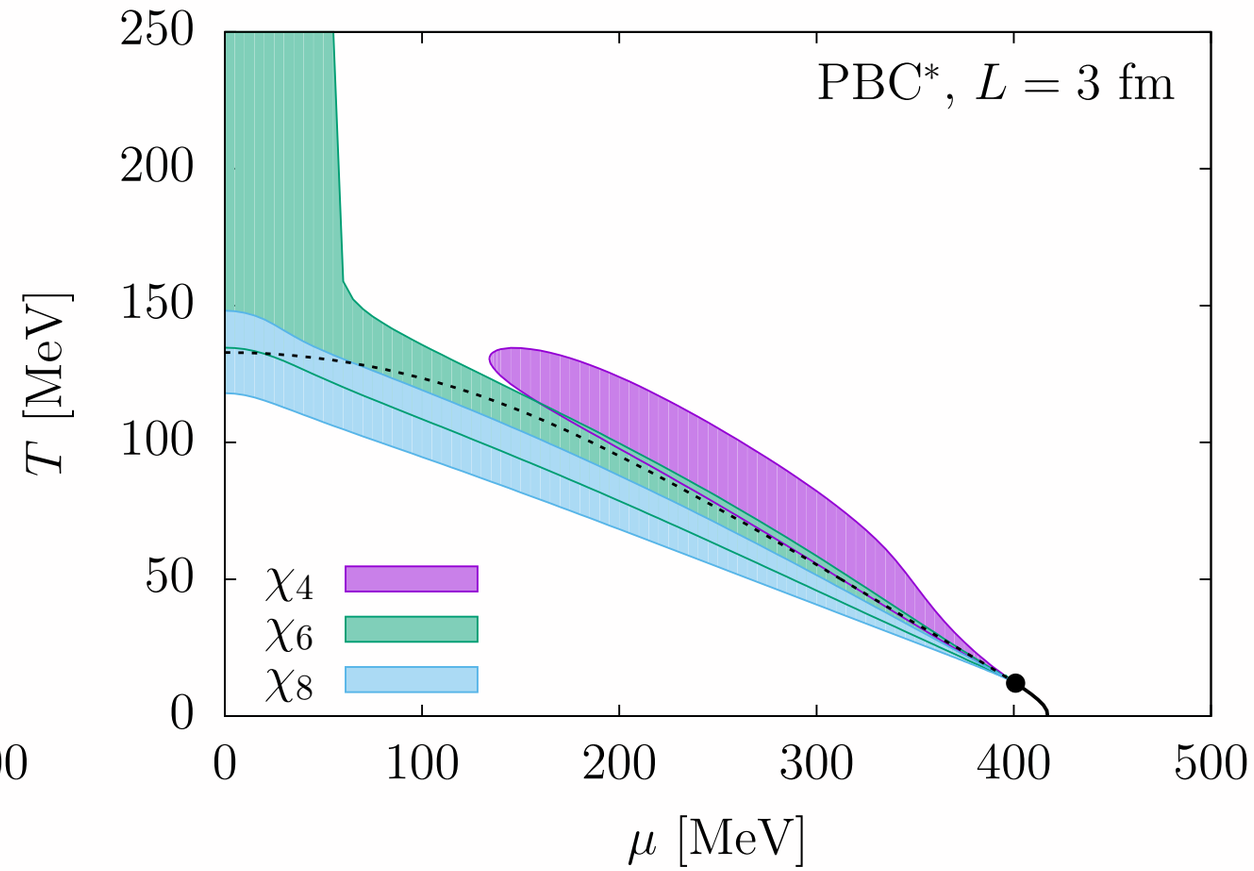




[A Juricic, BJS in preparation]

# Vacuum fluctuations

**renormalizes MFA (with vacuum fluctuations)**

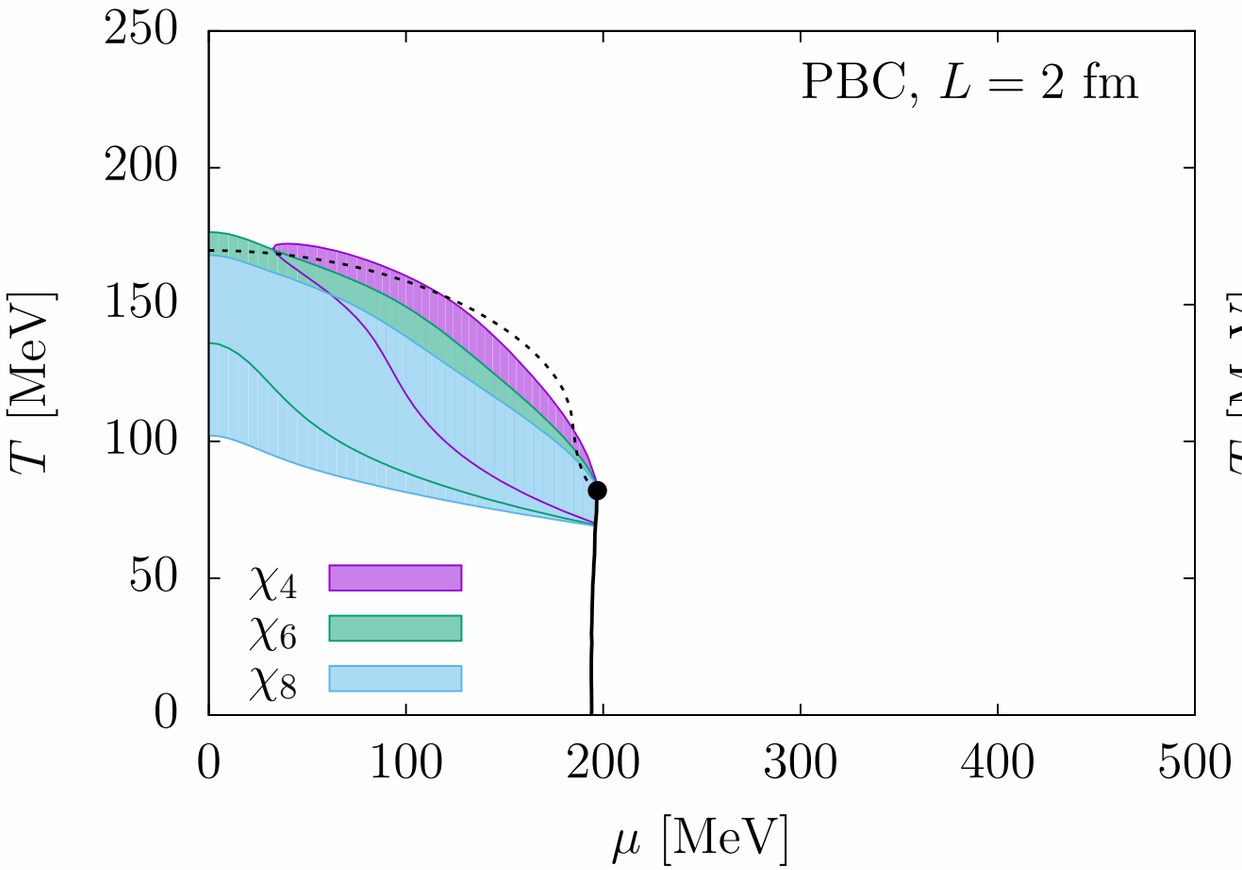


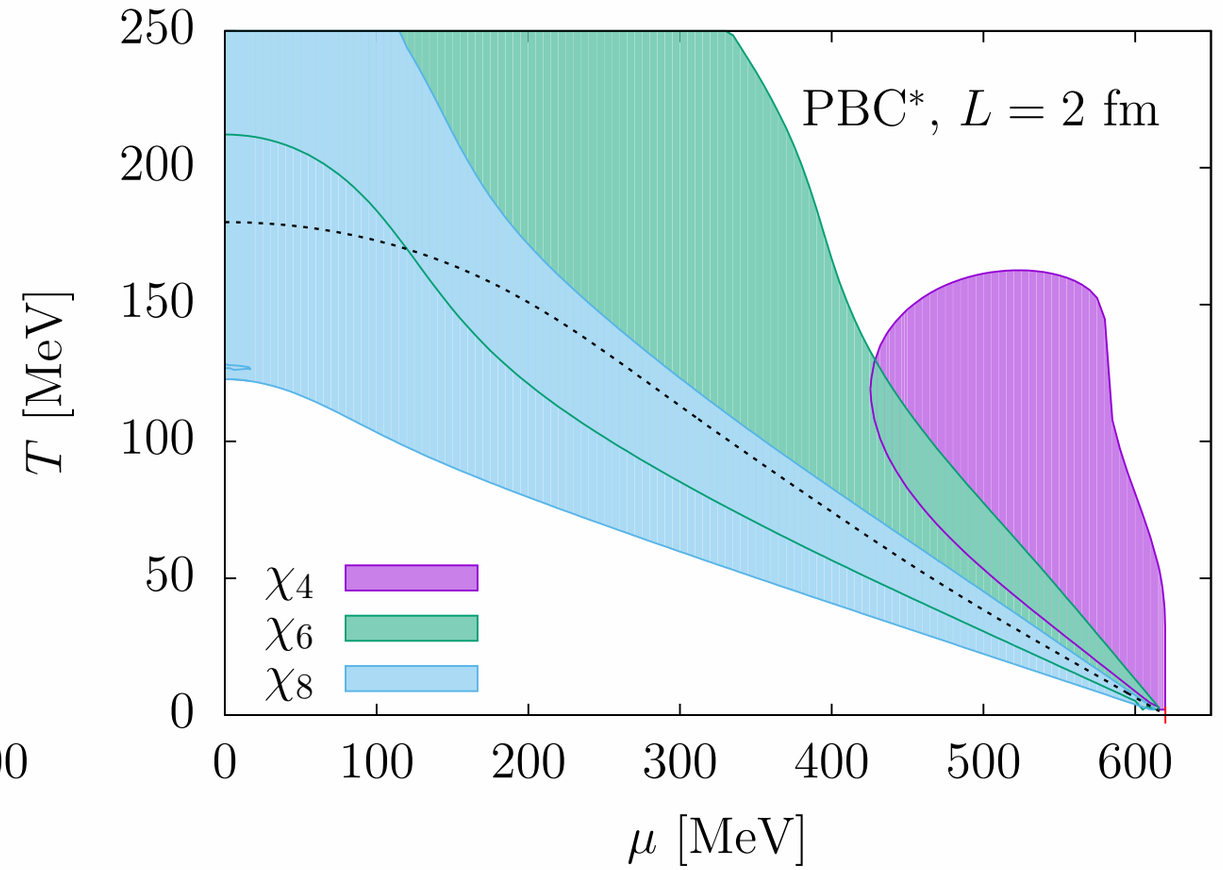




[A Juricic, BJS in preparation]

# Vacuum fluctuations

**renormalizes MFA (with vacuum fluctuations)**




[A Juricic, BJS in preparation]

# Summary & Conclusions

- Finite volume field theoretical investigation

- effects of quantum and thermal fluctuations
  fluctuations wash out the phase transition

- existence of critical points in phase diagram
  in finite volume

**Outlook: FRG in a finite volume**
**mesonic fluctuations**