Timelike charge pion form factor in DSEs framework

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Outline

- Introduction
- pion form factors
- DSEBSE model
- Results for BG RLA and beyond
- Conclusion

based on V.S arXiv:1505.03778 and V.S. PRD 90, (2014); V.S. PRD 86, (2012); V.S., P. Bicudo, QCD-TNTII (2011), V.S. IJTP 2015 arXiv:1411.2568

Introduction

QCD is theory of hadrons based on Quantum Field Theory of quarks and gluons.

PT QCD - hard inclusive processes

Exclusive processes.. ?

The basic problem of hadron physics is to solve QCD at all possible levels.

- effective preQCD VMD for pion and nucleon ff, CQM
- effective (after) QCD -ChPT, resonance ChPT
- QCD degrees of freedom lattice and DSEBSEs

Pion form factors

Recent progress in **photon pion transition f.f.** for $Q^2 < 0$

$< \pi(p)\gamma(q')|J_\delta(0)|0 >= -\epsilon_{\alpha\beta\nu\delta}\epsilon^{*\alpha}q^\beta q^{'\gamma}F_{\gamma,\pi}(q^2)$

$F_{\pi_0,\gamma} \leftrightarrow$ off shell pion ABJ triangle anomaly overlapped by other QCD structure

Experiments is complicated

spacelike f.f : QED Bha-Bha is bg.

$F_{\pi_0,\gamma}(Q^2) = F(Q_1^2 \simeq 0, Q^2 = Q_2^2)$

CLEO 1997, BABAR, Belle

Timelike f.f. QED anihilations to γ is bg.

Pion transition ff

SND CMD CMD2

$$e^+e^- \to 3\gamma \text{ such that } e^+e^- \to \pi\gamma \to 3\gamma$$

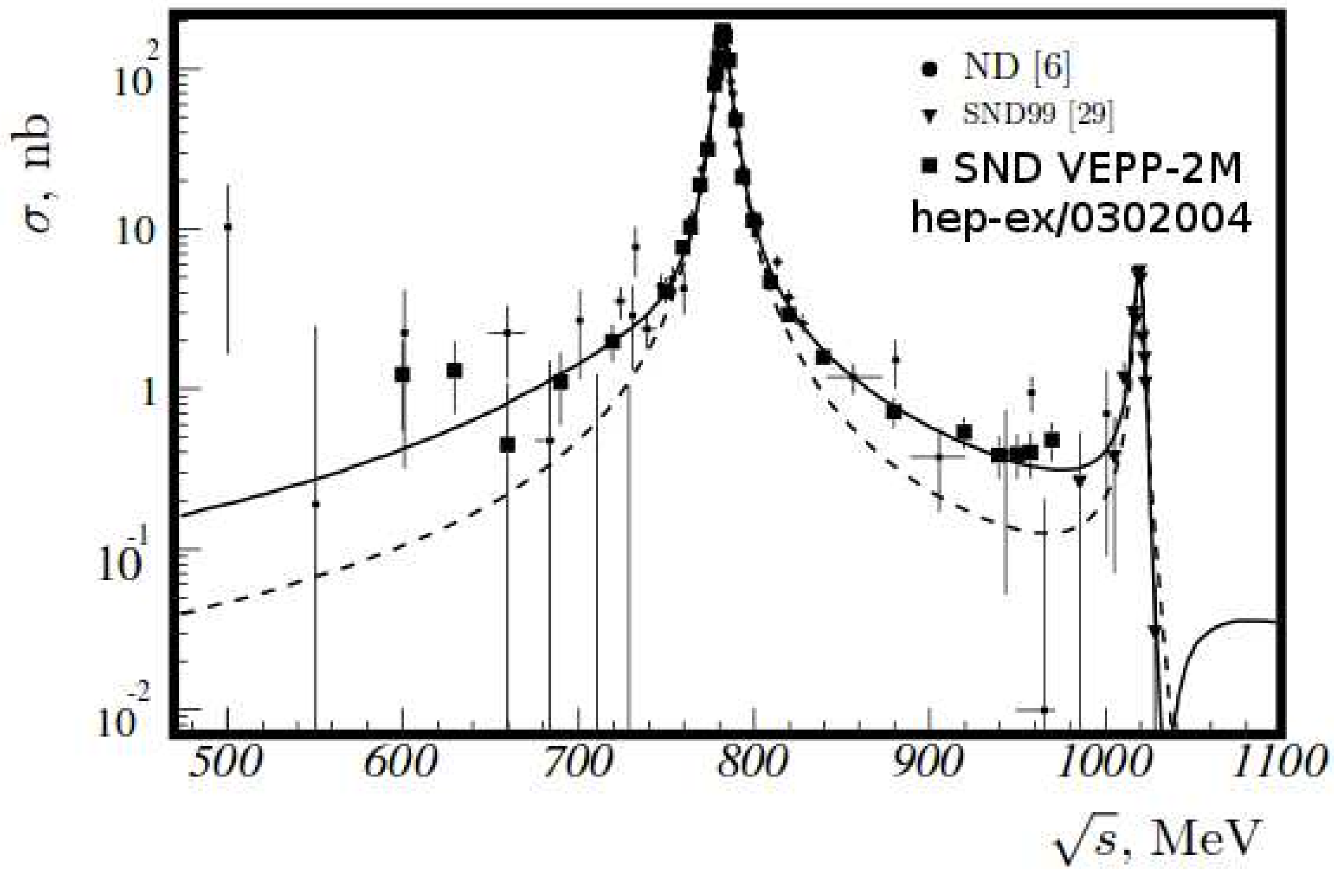
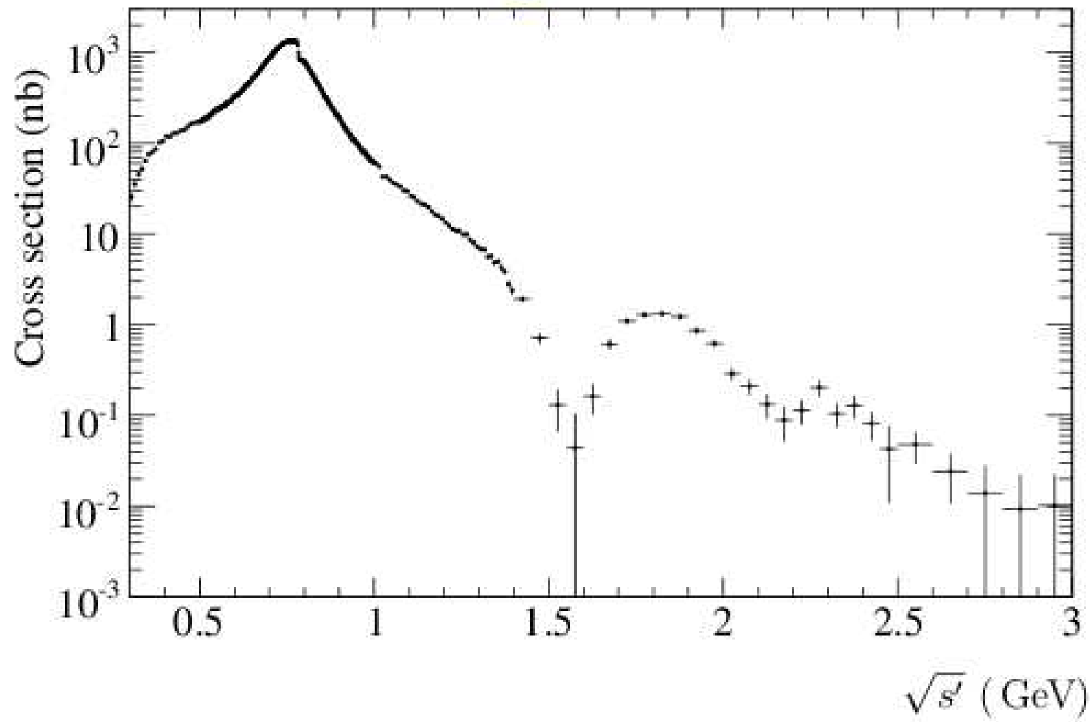


Figure 4: $e^+e^- \to \pi^0\gamma$ cross section. Solid line depicts the cross section in the

VMD. QED anihilations must be subtracted.

Pion charge form factor



$$F_\pi^2 = \frac{s}{3\beta_\pi^3 \alpha(0)} \sigma_{\pi\pi}(s)$$

BABAR 2012

Precise Measurement of the $e^+e^- \to \pi^+\pi^-(\gamma)$ Cross Section with the Initial-St
Radiation Method at *BABAR*

DSE

Most preferable theory for experimentalist is VMD

Our wish- Form factor in terms of QCD GF's

at the infrared domain one needs nonperturbative forms of GFs obtained from Dyson-Schwinger equations (DSEs)

Series of mesons are encoded in vertices

quark-hadron duality

e.g. ρ, ω and ϕ and their radial excitation are "hidden" in the transversal part quark-photon vertex

and one needs the solution for

Charge FF, RIA in Minkowski space

Pertubartion theory: G. Farrar and D. Jackson, PRL.(1979),G. P. Lepage, S. J. Brodsky , PL (1979), A.V. Efremov and A.V. Radyushkin, PL. (1980).

$$F_\pi(t) \rightarrow \frac{64\pi^2 f_\pi^2}{(11 - 2/3n_f)\, t\, \ln t/\Lambda^2_{QCD}}$$

no DR for positive ρ for F.

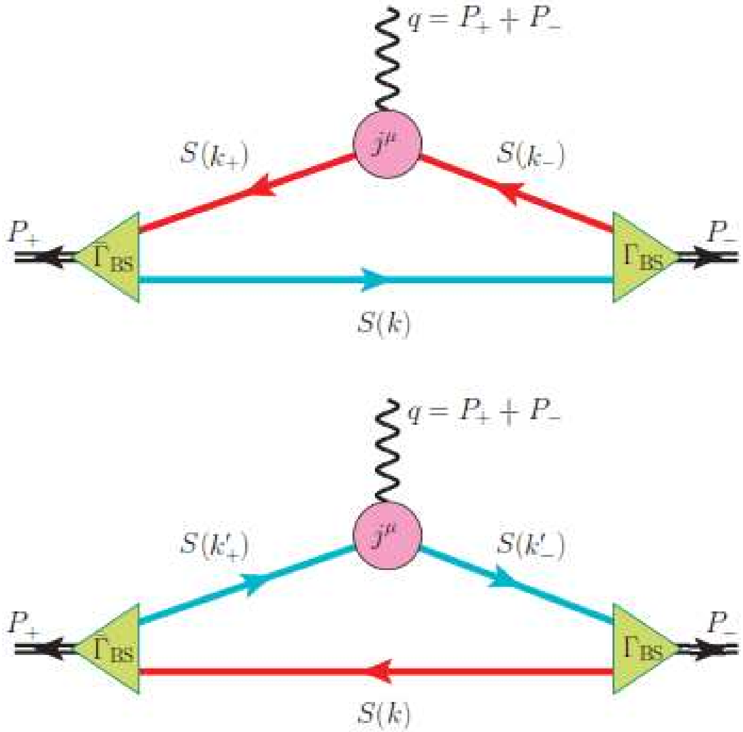


Figure 3: Two contributions to pion ff, up quark si blue

DSEBSE RIA: $G^\mu(P_+, P_-) = eF_\pi(q^2)(P_+ - P_-)^\mu$

$= \frac{2}{3} ie \int \frac{d^4k}{(2\pi)^4} tr \left[\tilde{\Gamma}_{BS}(k_+, P_+) S_u(k_1)\right.$

$\left.\Gamma^\mu_{EM}(k_1, k_2) S_u(k_2) \Gamma_{BS}(k_-, P_-) S(k_3)\right]$

$+ \ldots \frac{-1}{3} ie \ldots + \ldots$

Figure 3:

Background RIA

VMD - uses vector meson degrees of freedom

VM are encoded in quark-photon vertex, they are wide resonances => using homogeneous BSE is inappropriate

$$\Gamma_{EM} = \gamma + \int S\Gamma_{EM}SK \, .$$

Background approximation

$$\Gamma^{\mu}_{EM}(k_1, k_2) = \gamma_{\mu}$$

BA: skeleton 1-loop app. to which vector resonace couples. when full ansamble of Q-transverse vectors Γ_{EM} is convoluted with BA then one could get dual hadron picture.. UP to the phase!

Explicit evidence for interference of vertex with bg.. quark loops needs some phenomenology today

$$\Gamma^{\mu}_{EM}(q,q.p,p^2) = \gamma^{\mu}(1+\Gamma_{\rho}(q^2))$$

$$\Gamma_{\rho}(q^2) = \Sigma_{\rho_i} e^{i\phi_{EM}(q^2)} \frac{r_i q^2 m^2_{\rho_i}}{(q^2 - m^2_{\rho_i})^2 + \Gamma^2_{\rho_i} m^2_{\rho_i}}$$

where the quark gap equation phase

$$\phi_{EM1}(q^2) = f_q \frac{\sqrt{q^2/a^2}}{L(q^2/a^2)}$$

defines the first model (**RIA1**) with the damping log function L

The second model (**RIA2**) is completed by using the phase defined in Eq. (1)

$$\phi_{EM2}(q^2) = f_{q2} \sqrt{\frac{q^2/a^2}{ln\left(e + q^2/a^2\right)}} \, ,$$

DSEBSE model

Quark DSE

$$
\begin{aligned}
S^{-1}(p) &= Z_2 \not{p} - Z_4 m_q(\mu) - Z_1 \Sigma(p) \\
\Sigma(p) &= i \int \frac{d^4k}{(2\pi)^4} G_{\mu\nu}(p-k) \frac{\lambda^a}{2} \gamma_\mu S(k) \Gamma^a_\nu(p,q) \,,
\end{aligned}
$$

Pion BSE:

$$
\Gamma^j_{\pi n}(p,P) = i \int \frac{d^4k}{(2\pi)^4} K(k,p,P) \chi(k,P)^j_{\pi n}
$$

DSEBSE model

State of art of Minkowski space BSE solutions

working directly with Minkowski metric is possible!

(Note, numerics is not well defined problem in Minkowski for PT, one actually needs analytical continuation) however..

1. there are no poles at real axis for quark and gluon propagators

2. great improvement is achieved if an analytical integration over Minkowski 3d- spherical part can be done

3. when F is calculated, the analytical fit for S, Γ is used

Confinement allows direct treatment in Minkowski space

BSEDSE RL kernel:

$$
\begin{aligned}
K(x) &= \frac{\alpha(x)}{\pi x} = C_{if} K_{if}(x) + C_{uv} K_{uv}(x) \\
K_{if}(x) &\simeq -\Theta(-x) \frac{d}{dx} \left[\exp(x\mathcal{A}(-x)^{1/2} \ln \left(G^{-1}(x) \right) \right] \\
&- \Theta(x) \frac{d}{dx} \left[e^{i(x\mathcal{A}(x))^{1/2}} \ln \left(G^{-1}(x) \right) \right] \\
\mathcal{A}(x) &= \frac{1}{\ln \left[e + (x/a)^{1/2} \right]} \\
G^{-1}(x) &= \frac{(x - a^2) + a^4}{2a^4} \\
K_{uv}(x) &= \frac{d}{dx} \ln \ln \left(e^2 + \left[\frac{x}{\Lambda^2_{QCD}} \right]^2 \right)
\end{aligned}
$$

$$
\Gamma_i(p_0, p^2; P) = i\Sigma_j \int_{-\infty}^{\infty} dk_o \int_{-k^2}^{\infty} dk^2 \sqrt{k^2 - k_o^2} \int_{-1}^{1} dz f_j(k,^2, p^2, k.p, k.P)
$$

where z is

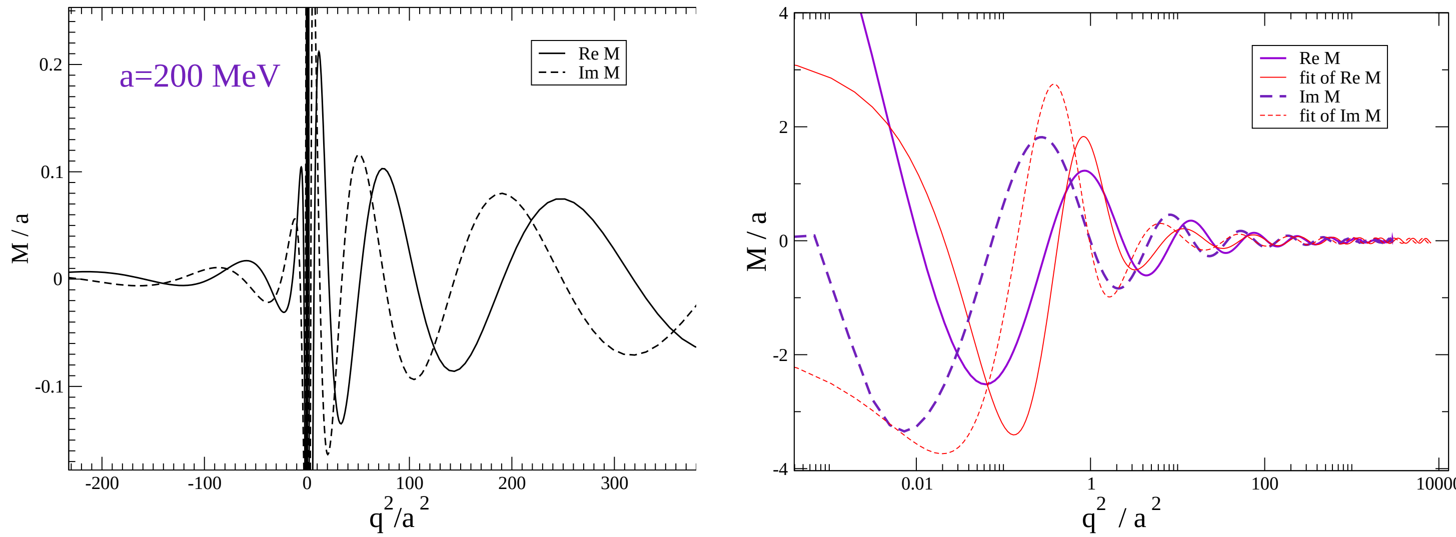


Figure 4: Solution of QCD gap equation: Left panel shows the quark dynamical mass function as a function of Minkowski square of momentum q^2 The same function and the analytical fit (**??**) are displayed on the right panel against lnq^2.

BG RIA (no vector meson)

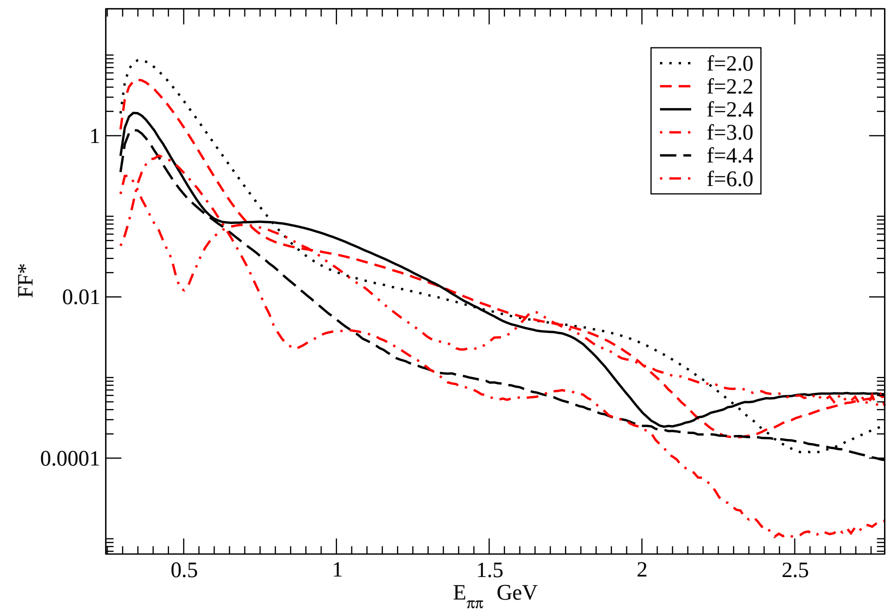


Figure 5: The **BRIA** pion form factor $F(Q^2)$ for combined kernel $\kappa = 1/2$ discussed in the Section III. The DSE solution gives $m_\pi/a = 0.2$, $(m_\pi = 139MeV)$ and the mass function fit $f_q \simeq 2.2$. The results for higher values $f_q = 3.0, 4.4$ and $f_q = 3, 6.0$ represent the pion form factor obtained with DSE/BSE solutions where $m_\pi/a = 0.3, 0.5$ and $m_\pi/a = 0.6$ respectively.

RIA phenomenology (one vector meson)

reminder of phenomenology:

$$\Gamma_\rho(q^2) = e^{i\phi_{EM}(q^2)} \frac{r q^2 m_\rho^2}{(q^2 - m_\rho^2)^2 + \Gamma_\rho^2 m_\rho^2}$$

$$\phi_{EM1}(q^2) = f_q \frac{\sqrt{q^2/a^2}}{L(q^2/a^2)}$$

with f_q, a known from quark DSE and $m_{rho} = 775 MeV$

The second model (**RIA2**) is completed by using the phase defined in Eq. (1)

$$\phi_{EM2}(q^2) = f_{q2} \sqrt{\frac{q^2/a^2}{ln\left(e + q^2/a^2\right)}} \,,$$

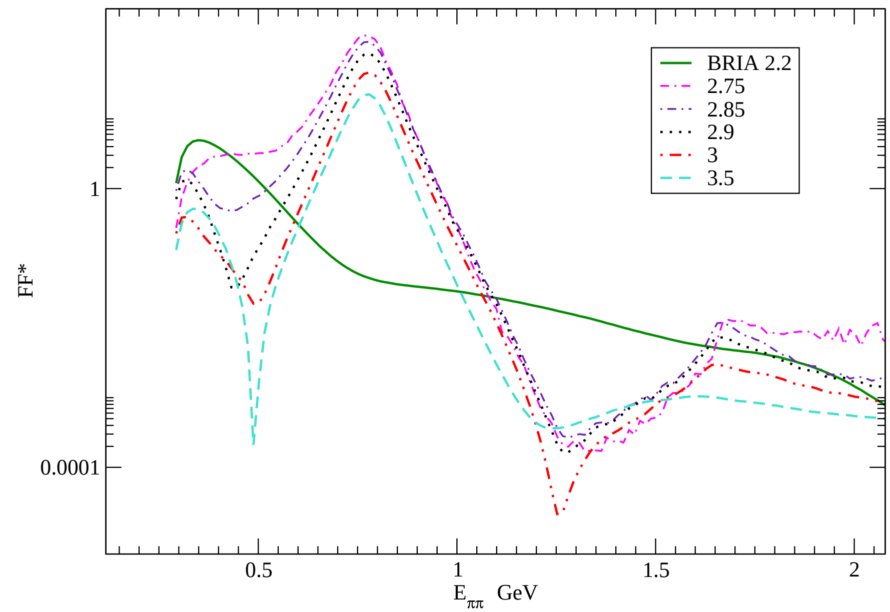


Figure 6: Line-shape of pion form factors as obtained in **RIA1** for various f_q in Eq. (1). $FF*$ **BRIA** is added for comparison. In presented model only $\rho(770)$ meson is considered, heavier resonances are not included and the dip as well as the second peak are pure interference effects.

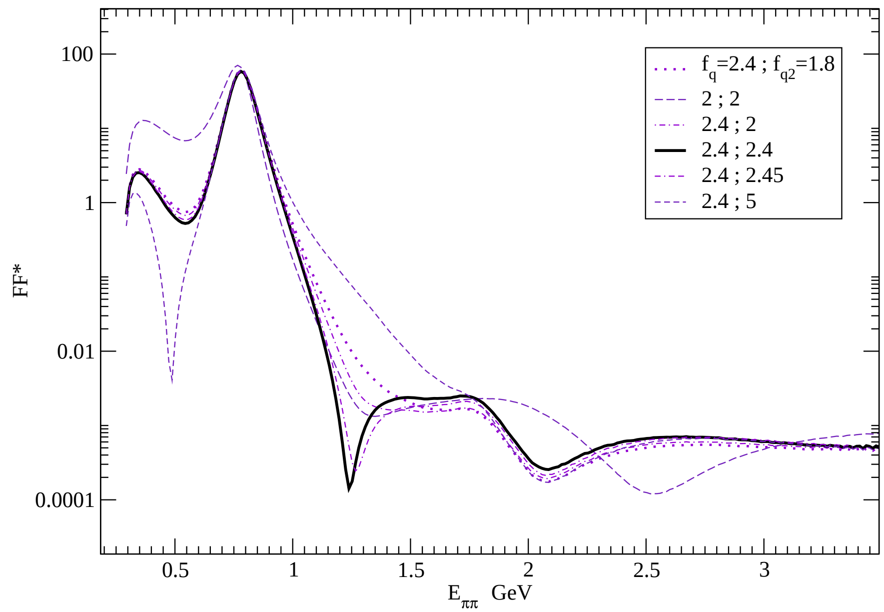


Figure 7: The pion form factors as obtained in **RIA2** for various f_q and f_{q2} in Eq. (1) with $\rho(770)$ included as described in the text.

RIA2 phenomenology (two vector mesons)

reminder of phenomenology:

$$\Gamma_\rho(q^2) = \Sigma_{\rho_i} e^{i\phi_{EM}(q^2)} \frac{r_i q^2 m_{\rho_i}^2}{(q^2 - m_{\rho_i}^2)^2 + \Gamma_{\rho_i}^2 m_{\rho_i}^2}$$

where the quark gap equation phase

$$\phi_{EM1}(q^2) = f_q \frac{\sqrt{q^2/a^2}}{L(q^2/a^2)}$$

with f_q, a known from quark DSE and $m_{\rho_1} = 775 MeV$ $m_{\rho_2} = 2300 MeV$

The second model (**RIA2**) is completed by using the phase defined in Eq. (1)

$$\phi_{EM2}(q^2) = f_{q2} \sqrt{\frac{q^2/a^2}{ln\left(e + q^2/a^2\right)}}\,,$$

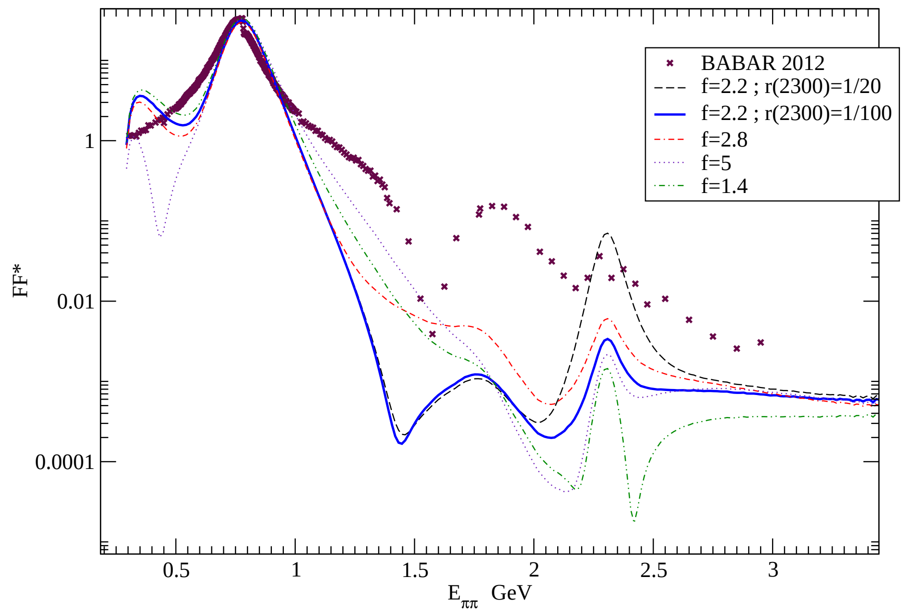


Figure 8: The pion form factors as obtained in **RIA2** for various f_q and f_{q2} with two ρ mesons added as described in the text. The dashed line is calculated with $r = 1/20$, the rest with $r = 1/100$. The experimental data are from [?].

Conclusion

The first results for the timelike pion form factor, which are calculated from QCD GFs (gluon and quark propagators)

Working for $\sqrt{s} < 5GeV$ for RIA2 (but only for $\sqrt{s} < 1.5or2GeV$ for RIA1)

DIP at 1500MeV is the effect of interference

CHPT and VMD analyticity is lost! No $2m_{pi}$ threshold is seen!

In any case, F is not direct observable, but the scalar product FF^* should be and the same physics will be certainly achieved when

$$F_{DSE/BSE}(q^2) = F_{phen}(q^2)e^{i\delta_C(q^2)} \, ,$$

where F_{phen} represents calculation based on ADS-CFT,ChPT,SQM,VMD or whatever phenomenology,

Future improvements

1. Problem of uniqueness, there are more solutions for GFS. When one has to combine with perturbative sector of SM one has to properly choose which one belongs to "$i\epsilon$ analytical continued perturbative part of SM.

2. Going beyond BG RIA. Solution of DSE for vertices, e.g. correct inclusion of transverse and longitudinal parts is necessary for $Q^2 > 0$

$$(q^\mu - \frac{Q^\mu q.Q}{Q^2})F_1(q,Q) \ ; \ Q^\mu \frac{M(p^2) - M(p'^2)}{p^2 - p'^2}$$

3. The other ff...