A non-perturbative BRST symmetry for the Gribov-Zwanziger action

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Outline

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What is the problem?

- Gribov has shown that the Faddeev-Popov procedure to fix a gauge does not eliminate all gauge redundancy.
- This implies we have equivalent gauge field configurations being integrated over in the path integral.
- Why our perturbative computations seem to be ok?

- Although they exist also for configurations related through finite gauge transformations, we restrict ourselves to the case of infinitesimal copies.
- In this case, it is possible to show that copies are related to zero-modes of the Faddeev-Popov operator.
- Therefore, elimination of (infinitesimal) copies $\Leftrightarrow$ elimination of zero-modes of the Faddeev-Popov operator.

- Possible (partial) solution to the problem: find a region Ω (the Gribov region) in field space which is free from zero-modes and restrict the path integral to it. *Gribov '78*

A "restriction" for the restriction

- In his paper, Gribov studied the problem in Landau gauge, *i.e.* $\partial_\mu A^a_\mu = 0$
- Singer showed that this is not a particular problem of Landau gauge, but a quite general feature. *Singer '78*
- The restriction of the path integral to the Gribov region, although suitable to any gauge in principle, suffers from a technical issue which is precisely the very definition of it.
- In this case, we have to work out the solution "gauge by gauge".

- Restricting to Landau gauge, the Faddeev-Popov operator

$$\mathcal{M}^{ab} \equiv -\partial_\mu(\delta^{ab}\partial^2 - gf^{abc}A^c_\mu\partial_\mu) \ \text{with} \ \partial_\mu A^a_\mu = 0 \tag{1}$$

is hermitian.

- Therefore, we can define a region where $\mathcal{M}$ is positive. This region Ω is the original Gribov region and is free from infinitesimal Gribov copies.

$$\Omega = \left\{\mathcal{M}^{ab} > 0 \mid \partial_\mu A^a_\mu = 0\right\} . \tag{2}$$

The modified path integral

- The Gribov region has very nice geometrical features: *(i) It is bounded in every direction, (ii) it is convex and (iii) All gauge orbits cross it. Dell'Antonio and Zwanziger '91*
- Therefore, it is a good candidate to implement the restriction of the path integral.

The path integral is formally written as

$$\mathcal{Z} = \int \mathcal{D}\Phi \mathcal{V}(\Omega) \mathrm{e}^{-S} \,, \tag{3}$$

where Φ denotes all fields of the theory, $\mathcal{V}(\Omega)$ is responsible for the restriction of the integration domain and S is defined by

$$S = S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_{\mathrm{ghosts}} + S_{\mathrm{sources}} \,. \tag{4}$$

The action free from (infinitesimal) copies

- Gribov and Zwanziger worked out an effective way to implement the restriction to Ω.
- It corresponds to the inclusion of an additive term in S.
- The new action is known as the Gribov-Zwanziger action and is given by

$$S_{\mathrm{GZ}} = S + \gamma^4 H - dV\gamma^4(N^2 - 1)\,, \tag{5}$$

where γ is the so-called Gribov parameter and H, the horizon function,

$$H(A) = g^2 \int d^d x d^d y \; f^{abc} A^b_\mu(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A^e_\mu(y)\,. \tag{6}$$

- The Gribov parameter is not free and is fixed by a gap equation,

$$\langle H(A)\rangle = dV(N^2 - 1)\,. \tag{7}$$

- The horizon function is non-local and, therefore, the Gribov-Zwanziger action is non-local!

Local Gribov-Zwanziger action

- Remarkably, the Gribov-Zwanziger action can be cast in local form by the introduction of auxiliary fields.
- These fields also form a BRST quartet.

- We introduce a pair of bosonic $(\varphi^{ab}_{\mu}, \bar{\varphi}^{ab}_{\mu})$ and fermionic fields $(\omega^{ab}_{\mu}, \bar{\omega}^{ab}_{\mu})$.
- The local Gribov-Zwanziger action is

$$S_{\mathrm{GZ}} = S + \int d^d x \, \left(\bar{\varphi}^{ac}_{\mu}\mathcal{M}^{ab}\varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu}\mathcal{M}^{ab}\omega^{bc}_{\mu} + g\gamma^2 f^{abc}A^a_{\mu}(\varphi+\bar{\varphi})^{bc}_{\mu}\right) - dV\gamma^4(N^2-1)\,. \tag{8}$$

- This action is local and renormalizable to all orders in perturbation theory. *Zwanziger '89*

BRST *soft* breaking

- It is possible to show that in the UV, $\gamma \to 0$ and the usual Yang-Mills action in Landau gauge emerges.

- However, in the presence of γ, the Gribov-Zwanziger action is not invariant under the usual BRST transformations, namely

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c\,, \\
s\bar{c}^a &= b^a\,, & sb^a &= 0\,, \\
s\varphi^{ab}_\mu &= \omega^{ab}_\mu\,, & s\omega^{ab}_\mu &= 0\,, \\
s\bar{\omega}^{ab}_\mu &= \bar{\varphi}^{ab}_\mu\,, & s\bar{\varphi}^{ab}_\mu &= 0\,.
\end{aligned}
\tag{9}
$$

and

$$
sS_{\mathrm{GZ}} = \gamma^2 \int d^d x \, \left(g f^{abc} D^{ae}_\mu c^e (\bar{\varphi}^{bc}_\mu + \varphi^{bc}_\mu) + g f^{abc} A^a_\mu \omega^{bc}_\mu \right) \,. \tag{10}
$$

BRST *soft* breaking

- We see for $\gamma \to 0$, the breaking vanishes. Therefore, in the UV, the action is BRST invariant, as expected.

- At present, a full understanding of the BRST breaking is still lacking. *Cucchieri et al., Lavrov et al., Sorella et al., Zwanziger et al.*

- In this framework, the non-perturbative regime of Yang-Mills theories is characterized by a soft breaking of the standard BRST symmetry.
- Intuitively, the reason is that at the non-perturbative level, the effects of the Gribov copies are stronger and a non-trivial manifestation of the horizon of Ω enters the game.

Is it possible to reconcile BRST with the Gribov horizon?

Gauge invariant A^h field

- Let us consider the transverse field A^h, $\partial_\mu A^h_\mu = 0$, obtained by the minimization of

$$\int d^d x \, A^a_\mu A^a_\mu \,. \tag{11}$$

- This field is gauge invariant order by order in g and can be formally written as

$$A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right]\right) + O(A^3)\,. \tag{12}$$

- Its gauge invariance implies $sA^h = 0$.
- The form of the horizon function $H(A)$ and of A^h allows us to write the following expression

$$H(A) = H(A^h) - R(A)(\partial A)\,, \tag{13}$$

where $R(A)(\partial A) = \int d^d x d^d y \; R^a(x,y)(\partial A^a)_y$.

The "new" Gribov-Zwanziger action

- The Gribov-Zwanziger action is rewritten as

$$\tilde{S}_{\rm GZ} = S_{\rm YM} + \int d^dx \left(b^{h,a}\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b\right) + \gamma^4 H(A^h)\,, \tag{14}$$

with

$$b^{h,a} = b^a - \gamma^4 R^a(A)\,. \tag{15}$$

- As did before, we introduce the localizing auxiliary fields. The resulting action is

$$\begin{aligned} S_{\rm GZ} &= S_{\rm YM} + \int d^dx \left(b^{h,a}\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b\right) \\ &+ \int d^dx \left(\bar{\varphi}^{ac}_\mu \left[\mathcal{M}(A^h)\right]^{ab}\varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \left[\mathcal{M}(A^h)\right]^{ab}\omega^{bc}_\mu + g\gamma^2 f^{abc}A^{h,a}_\mu(\varphi+\bar{\varphi})^{bc}_\mu\right). \end{aligned} \tag{16}$$

The *non-perturbative* BRST symmetry

- We define an operator s_{γ^2}, as follows

$$
\begin{aligned}
s_{\gamma^2} A^a_\mu &= -D^{ab}_\mu c^b \,, & s_{\gamma^2} c^a &= \frac{g}{2} f^{abc} c^b c^c \,, \\
s_{\gamma^2} \bar{c}^a &= b^{h,a} \,, & s_{\gamma^2} b^{h,a} &= 0 \,, \\
s_{\gamma^2} \varphi^{ab}_\mu &= \omega^{ab}_\mu \,, & s_{\gamma^2} \omega^{ab}_\mu &= 0 \,, \\
s_{\gamma^2} \bar{\omega}^{ab}_\mu &= \bar{\varphi}^{ab}_\mu + \gamma^2 g f^{cdb} A^{h,c}_\mu \left[\mathcal{M}^{-1}(A^h)\right]^{da} \,, & s_{\gamma^2} \bar{\varphi}^{ab}_\mu &= 0 \,.
\end{aligned} \tag{17}
$$

- This operator satisfies the relations

$$
s_{\gamma^2} = s + \delta_{\gamma^2} \quad \text{and} \quad s^2_{\gamma^2} = 0 \tag{18}
$$

  and defines a symmetry of the Gribov-Zwanziger action, namely,

$$
s_{\gamma^2} S_{\mathrm{GZ}} = 0 \,. \tag{19}
$$

- In the limit $\gamma \to 0$, $s_{\gamma^2} \to s$.
- Therefore, the standard (or *perturbative*) BRST transformations are "corrected" through the introduction of non-perturbative terms.
- In this sense, we speak about a *non-perturbative* BRST symmetry.

Non-perturbative *Vs.* perturbative BRST

- A non-perturbative Slavnov-Taylor identity implies

$$\langle s_{\gamma^2}(\bar{c}\Lambda)\rangle = 0 \quad \Rightarrow \quad \langle s(\bar{c}\Lambda)\rangle = -\langle \delta_{\gamma^2}(\bar{c}\Lambda)\rangle \,. \tag{20}$$

- This relation shows that the *perturbative* BRST operator is associated with a breaking which is proportional to the Gribov parameter γ.

- With the non-perturbative operator s_{γ^2}, we can propose a "*non-perturbative*" BRST quantization.

Linear covariant gauges

- Recently, different groups from different approaches are trying to extend their results from Landau gauge to linear covariant gauges.
- This class of gauges contain a gauge parameter α and is defined by

$$\partial_\mu A^a_\mu = \alpha b^a \,. \tag{21}$$

- In this case, the construction of a Gribov region is not as clear as in the Landau gauge.
- For general α (different from zero), the Faddeev-Popov operator is not hermitian.
- Some attempts to define a suitable region to restrict the path integral were done, but such region is not well understood as Ω. *Sobreiro and Sorella '05; Capri, Pereira, Sobreiro and Sorella '15.*

- Using the non-perturbative BRST operator we propose a Gribov-Zwanziger action which is invariant under the modified BRST transformations.
- Afterwards we try to understand the geometrical interpretation of such action (in the sense of the restriction to a suitable region)

Gribov-Zwanziger action in linear covariant gauges

- The Gribov-Zwanziger action in linear covariant gauges is constructed as

$$
\begin{aligned}
S_{\rm GZ}^{\rm LCG} &= S_{\rm YM} + s_{\gamma^2}\int d^dx\, \bar{c}^a\left(\partial_\mu A^a_\mu - \frac{\alpha}{2}b^{h,a}\right) \\
&+ \int d^dx\left(\bar{\varphi}^{ac}_\mu\left[\mathcal{M}(A^h)\right]^{ab}\varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu\left[\mathcal{M}(A^h)\right]^{ab}\omega^{bc}_\mu + g\gamma^2 f^{abc}A^{h,a}_\mu(\varphi+\bar{\varphi})^{bc}_\mu\right) \\
&= S_{\rm YM} + \int d^dx\left(b^{h,a}\left(\partial_\mu A^a_\mu - \frac{\alpha}{2}b^{h,a}\right) + \bar{c}^a\partial_\mu D^{ab}_\mu c^b\right) \\
&+ \int d^dx\left(\bar{\varphi}^{ac}_\mu\left[\mathcal{M}(A^h)\right]^{ab}\varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu\left[\mathcal{M}(A^h)\right]^{ab}\omega^{bc}_\mu + g\gamma^2 f^{abc}A^{h,a}_\mu(\varphi+\bar{\varphi})^{bc}_\mu\right)
\end{aligned} \tag{22}
$$

- This action is invariant under non-perturbative BRST transformations, namely,

$$
s_{\gamma^2} S_{\rm GZ}^{\rm LCG} = 0\,. \tag{23}
$$

- In the limit $\alpha \to 0$, the previous action reduces to the Gribov-Zwanziger in Landau gauge.

Dependence on α

- An important fact is the computation of the gap equation. It can be done as follows

$$\frac{\partial \mathcal{E}_v}{\partial \gamma^2} = 0 \quad \Rightarrow \quad \langle H(A^h) \rangle = dV(N^2 - 1)\,, \tag{24}$$

where

$$\mathrm{e}^{-V\mathcal{E}_v} = \int [\mathcal{D}\Phi]\, \mathrm{e}^{-(S^{\mathrm{LCG}}_{\mathrm{GZ}} - dV\gamma^4(N^2-1))}\,. \tag{25}$$

- The gap equation is manifestly gauge invariant.
- Since this equation is used to determine the Gribov parameter, it implies γ is independent from α.
- The Gribov parameter enters physical quantities and, therefore, being independent from α is a crucial check.

Geometrical interpretation

- The action $S_{\mathrm{GZ}}^{\mathrm{LCG}}$ implements the restriction of the path integral to the region

$$\Omega^h = \left\{ \partial_\mu A^a_\mu = \alpha b^a \,, \; \partial_\mu A^{h,a}_\mu = 0 \,, \; \mathcal{M}(A^h) > 0 \right\} \,. \tag{26}$$

- In the limit $\alpha \to 0$, $\Omega^h \to \Omega$.
- The region Ω^h is convex and bounded in all directions.
- These properties follow from the transversality of A^h.

- It is possible to show that for regular functions $\xi^a(x;\alpha)$, the condition $\mathcal{M}(A^h) > 0$ implies $\mathcal{M}^{ab}(A)\xi^b \neq 0$.

Gluon propagator

- At tree level,

$$A^h \approx A_\mu - \frac{\partial_\mu}{\partial^2}(\partial A) \equiv A^T_\mu \,. \tag{27}$$

- The gluon propagator is

$$\langle A^a_\mu(k) A^b_\nu(-k)\rangle = \delta^{ab}\left[\frac{k^2}{k^4 + 2g^2\gamma^4 N}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{\alpha}{k^2}\frac{k_\mu k_\nu}{k^2}\right]\,. \tag{28}$$

- The longitudinal part does not receive non-perturbative corrections. It is possible to show this property holds to all orders - *In the usual BRST soft breaking, this property does not necessarily hold.*
- The transverse part is identical to Landau gauge propagator in the Gribov-Zwanziger framework.

Perspectives

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- The formulation, although BRST invariant, is non-local. The natural step is to cast all framework in local form. *Work in progress*
- A complete understanding of Ω^h is important in order to characterize how general is this construction.
- The "non-perturbative" quantization procedure can be extended to other gauges even more complicated, as non-linear gauges.
- The invariance under non-perturbative BRST is compatible with the Refined Gribov-Zwanziger scenario. A set of non-perturbative BRST transformations which contemplates the condensates masses is viable and all important properties remain valid. *To appear.*

Conclusions

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- A reformulation of the Gribov-Zwanziger action in Landau gauge with gauge invariant variables was proposed.
- A new set of BRST transformations was constructed. These transformations correspond to a symmetry of the Gribov-Zwanziger action.
- The new BRST symmetry feels the restriction of the path integral to the Gribov region and contains the non-perturbative parameter γ.
- A non-perturbative BRST quantization is proposed and the framework was naturally extended to linear covariant gauges.
- The reformulation puts the Gribov parameter as a manifestly gauge invariant.
- At this level, cohomology tools are at our disposal to define physical objects.

We hope this new proposal will bring new insights to the Gribov-Zwanziger framework and in non-perturbative approaches in general!

Thank You!