Scalar mass stability bound in a simple Yukawa-theory from renormalisation group equations

(arXiv:1508.06774)

Antal Jakovác, István Kaposvári, András Patkós

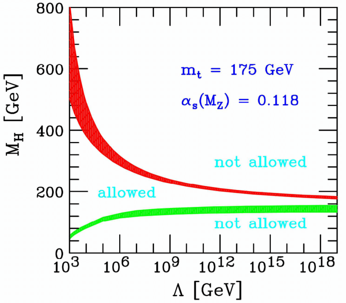
Department of Atomic Physics
Eötvös University, Budapest

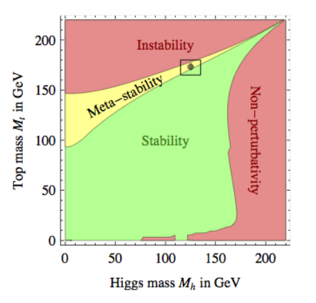
ACHT, 2015

Motivation

Stability Higgs bounds with perturbative RGE in SM.

$$\frac{d\lambda(Q^2)}{d\log Q^2} = \frac{1}{16\pi^2}\left(12\lambda^2 + 6\lambda h_t^2 - 3h_t^4 + \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)\right)$$





Non-perturbative studeis in simplified models:

Lattice: Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. Schroeder, 2007
D.Y.-J. Chu, K. Jansen, B. Knippschild, C.-J. D. Lin and A. Nagy, 2015

FRG: H. Gies, C. Gneiting and R. Sondenheimer, 2014

Outline

Higgs-top model with discrete chiral symmetry

Functional Renormalization Equations of the system

The Local Potential Approximation (LPA)

Consistent solutions

Results

Effective action of the Higgs-top toy model

Classical action of a scalar-fermion Yukawa bound system:

$$S = \int d^4x \left[\frac{1}{2}(\partial_\mu \sigma)^2 + U(\rho) + \bar{\psi} i \partial\!\!\!/ \psi + i h_k \sigma \bar{\psi}\psi \right], \quad \rho = \frac{1}{2}\sigma^2, \quad I = \sigma \bar{\psi}\psi$$

σ and I are invariant under the discrete chiral symmetry:

$$\sigma(x) \to -\sigma(x), \qquad \psi(x) \to \gamma_5 \psi(x), \qquad \bar{\psi}(x) \to -\gamma_5 \bar{\psi}(x)$$

Scale-dependent effective-action for $k < \Lambda$:

$$\Gamma_k = \int d^4x \left[\frac{Z_{\sigma,k}}{2}(\partial_\mu \sigma)^2 + U_k(\rho) + Z_{\psi,k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \sigma \bar{\psi}\psi \right], \quad \rho = \frac{Z_{\sigma,k}}{2}\sigma^2$$

Infrared observables:

$$v = Z_{\sigma 0}^{1/2} \sigma_0 = 246 GeV, \qquad m_\psi = h_0 \sigma_0 = 173 GeV$$

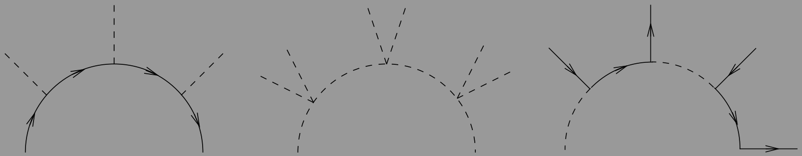
$$m_\sigma^2 = U_0'(\rho_0) + 2\rho_0 U_0''(\rho_0)$$

The Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\,\partial_t R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} = \frac{1}{2}\hat{\partial}_t \mathrm{STr}\log(\Gamma_k^{(2)} + R_k)$$

The right hand side of the Wetterich equation:

$$\frac{1}{2}\mathrm{STr}\log(\Gamma_k^{(2)} + R_k) = -\frac{1}{2}\mathrm{Tr}\log(\Gamma^{(2)}_{\Psi^T\Psi} + R_k^F) + \frac{1}{2}\mathrm{Tr}\log(\Gamma^{(2)}_{\sigma\sigma} + R_k^B)$$
$$+\frac{1}{2}\mathrm{Tr}\log\left[1 - \left(\Gamma^{(2)}_{\sigma\sigma} + R_k^B\right)^{-1}\Gamma^{(2)}_{\sigma\Psi}\left(\Gamma^{(2)}_{\Psi^T\Psi} + R_k^F\right)^{-1}\Gamma^{(2)}_{\Psi^T\sigma}\right]$$



ψ-loop in σ-bgd σ-loop in σ-bgd mixed loop in ψ-bgd

The Local Potential Approximation (LPA)

Constant background values, no field renormalization:

$$\sigma(x) \to v_k, \quad \psi(x) \to \psi_k, \quad \bar{\psi}(x) \to \bar{\psi}_k, \quad Z_{\sigma,k} \to 1, \quad Z_{\psi,k} \to 1$$

$$\partial_k [U_k(\rho_k) + h_k I_k] = \frac{1}{2} \hat{\partial}_k \int_q \Bigg[-4 \log(q_R^2 + m_\psi^2) + \log(q_R^2 + m_\sigma^2) + \\ + \log \left\{ 1 - h_k^2 \frac{1}{q_R^2 + m_\sigma^2} \frac{2 h_k I_k}{q_R^2 + m_\psi^2} \right\} \Bigg].$$

$$m_\sigma^2 = U_k'(\rho_k) + 2\rho_k U_k''(\rho_k), \qquad m_\psi^2 = 2 h_k^2 \rho_k$$

At a given scale ρ_k and I_k are connected by the equation of motion of the scalar field:

$$\sigma \frac{\delta \Gamma_k}{\delta \sigma}\Big|_{\sigma = v_k, \psi = \psi_k} = h_k I_k + 2\rho_k U_k'(\rho_k) = 0.$$

Consistent solutions

The consistency of the two sides of the LPA equation can be ensured in different ways.

Version *A*: Complete elimination of I_k on RHS

$$\partial_k h_t = 0,$$

$$\partial_t U_k(\rho_k) = \frac{1}{2}\hat{\partial}_t \int_q \Bigg\{ -5\log(q_R^2 + m_\psi^2) + \\ + \log\left[(q_R^2 + m_\sigma^2)(q_R^2 + m_\psi^2) + 4h_k^2\rho_k U'(\rho_k)\right] \Bigg\}.$$

Consistent solutions

Version B: Linearization of RHS in I_k

$$\partial_t (h_k I_k) = -\frac{1}{2}\hat{\partial}_t \int_q \frac{2h_k^3 I_k}{(q_R^2 + m_\psi^2)(q_R^2 + m_\sigma^2)},$$

$$\partial_t U_k(\rho_k) = \frac{1}{2}\hat{\partial}_t \int_q \Bigg[-4\log(q_R^2 + m_\psi^2) + \log(q_R^2 + m_\sigma^2) + \\ + \log\left\{ 1 - h_k^2 \frac{1}{q_R^2 + m_\sigma^2} \frac{2h_k I_k}{q_R^2 + m_\psi^2} \right\} \Bigg] \\ - \frac{1}{2}\hat{\partial}_t \int_q \frac{4h_k^2 \rho_k U_k'(\rho_k)}{(q_R^2 + m_\psi^2)(q_R^2 + m_\sigma^2)}.$$

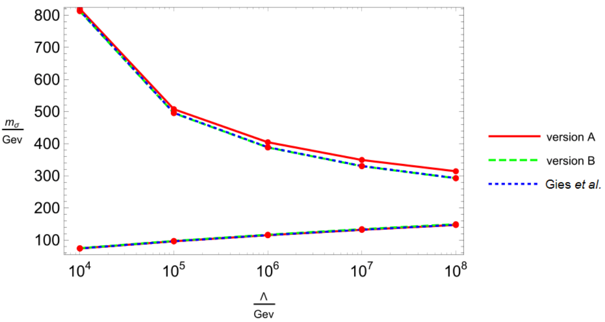
Gies *et al.* (2014): no ψ background, last two terms missing.

Results

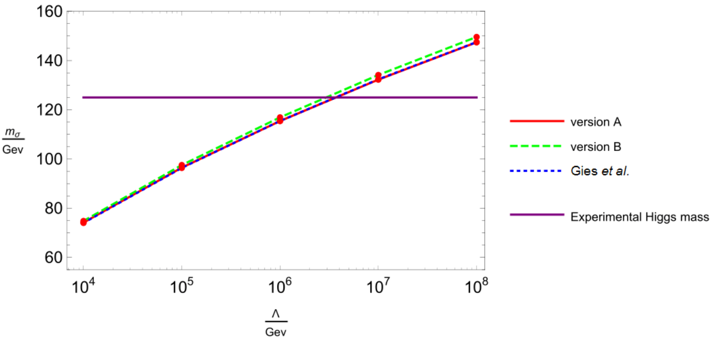
The potential used in the study for the symmetric (SYM) and symmetry broken (SB) regime respectively:

$$U_k^{(\mathrm{SYM})}(\rho_k) = \sum_{n=1}^{N_p} \frac{\lambda_n \rho_k^n}{n!}, \qquad U_k^{(\mathrm{SB})}(\rho_k) = \sum_{n=2}^{N_p} \frac{\lambda_n (\rho_k - \kappa_k)^n}{n!}$$

Higgs mass stability and triviality bounds for $N_p = 2$:



Results



Including Z_ψ, $Z_\sigma \neq 1$ leads to a percent level changes.
The maximum allowed value of the cutoff

$$N_p = 2: \qquad 2.9 \times 10^6 \text{GeV} < \Lambda_{max}^{(2)} < 3.7 \times 10^6 \text{GeV}$$

$$N_p = 4: \qquad 3.7 \times 10^6 \text{GeV} < \Lambda_{max}^{(4)} < 5.3 \times 10^6 \text{GeV}$$

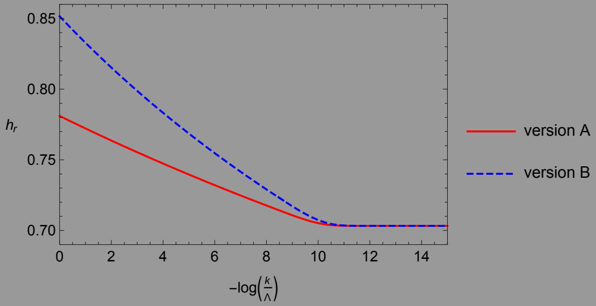
Summary

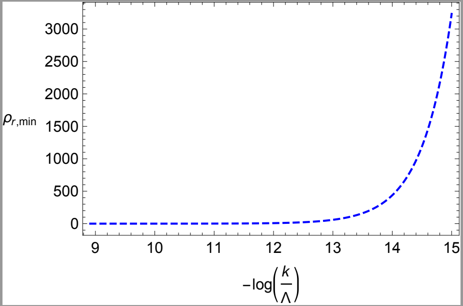
- The allowed range of the Higgs mass has been determined with FRG in presence of a pointlike composite fermion background.
- Close agreement of all approximation signals a robust determination of Λ_{max}

Outlook

- Investigation of a more general ansatz for $\Gamma[\rho, I]$
- Inclusion of the multiplet structure and the gauge interactions of the SM

Thank you for your attention!


0.85
0.80
0.75
0.70
h_r
0
2
4
6
8
10
12
14
$-\log\left(\frac{k}{\Lambda}\right)$
version A
version B


3000
2500
2000
1500
1000
500
0
$\rho_{r,\min}$
9
10
11
12
13
14
15
$-\log\left(\frac{k}{\Lambda}\right)$