Three-dimensional Yang-Mills theory as a testbed for truncations of Dyson-Schwinger equations

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ACHT2015, Leibnitz

October 8, 2015

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From Green functions to 'observables'

Basic building blocks of functional equations: n-point functions $\Gamma_{i_1 \dots i_n}$

Effective action: generating functional of 1PI Green functions

$$\Gamma[\Phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi_1 \dots \Phi_n \Gamma_{i_1 \dots i_n}$$

$\longleftrightarrow$

The set of **all** Green functions describes the theory completely.

$$\Gamma_{ij} = \frac{\delta^2 \Gamma[\Phi]}{\delta \Phi_i \delta \Phi_j},$$

$$\Gamma_{ijk} = \frac{\delta^3 \Gamma[\Phi]}{\delta \Phi_i \delta \Phi_j \delta \Phi_k}, \quad \dots$$

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Green functions $\rightarrow$ 'observables'?

Examples:

- Bound state equations $\rightarrow$ masses and properties of hadrons
  $\rightarrow$ talks of Hilger, Krassnigg, Sanchez-Alepuz, Sauli, ...
- Analytic properties of Green functions $\rightarrow$ confinement
- (Pseudo-)Order parameters $\rightarrow$ Phases and transitions
  $\rightarrow$ talk of Reinosa

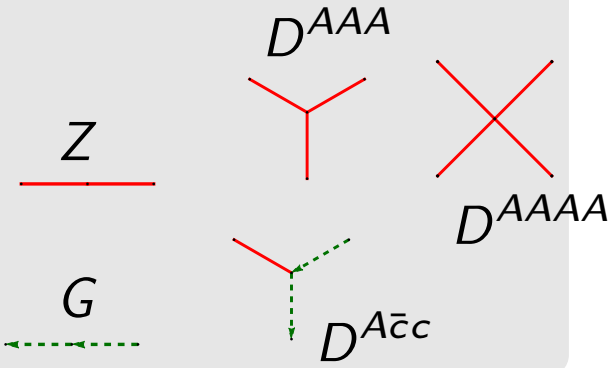
Landau gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

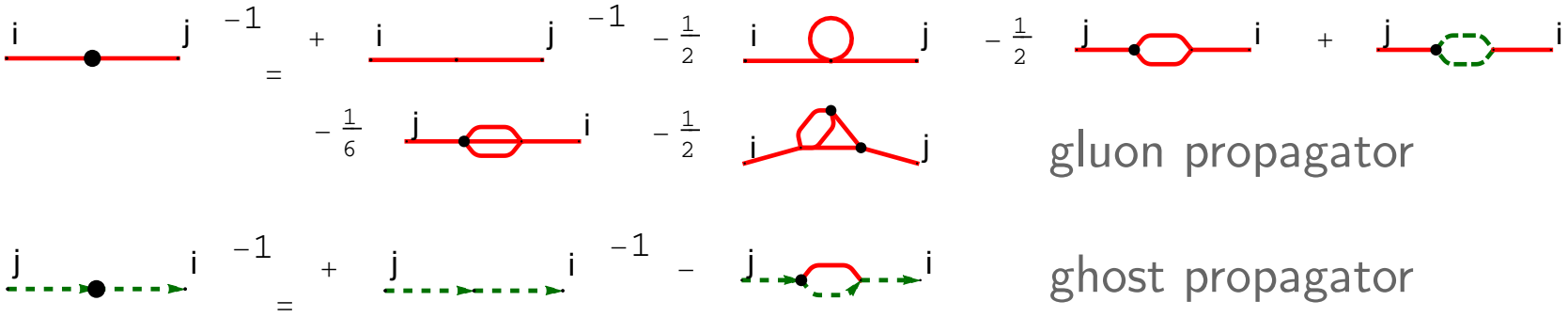
$$\mathcal{L} = \frac{1}{2} F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}} \left(-\Box + g\, \boldsymbol{A} \times \right) \boldsymbol{c}$



The tower of DSEs



gluon propagator

ghost propagator

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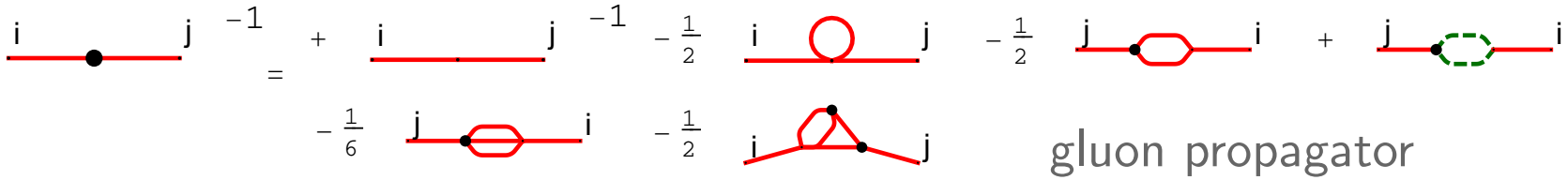

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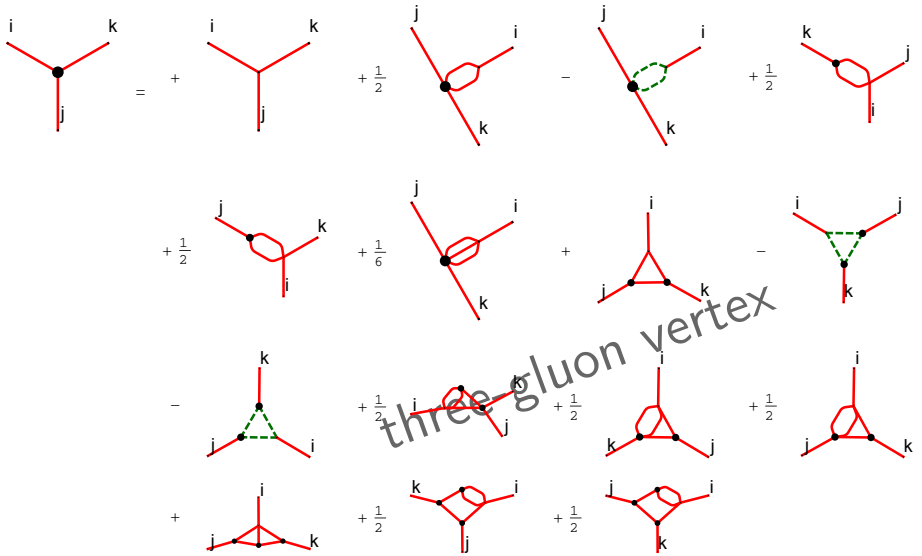

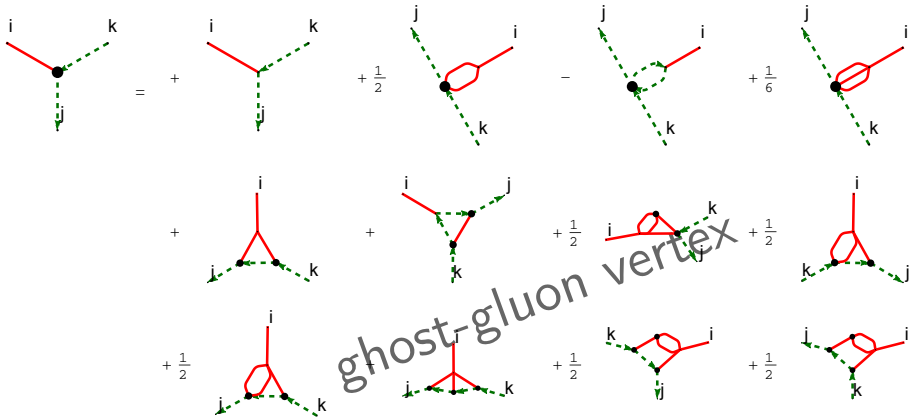

Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Truncating the equations

Truncation

- Drop quantities (unimportant?)
- Model quantities (good models available?)
- Use fits

Ideally: Find a truncation that has no parameters and yields quantitative results.

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Guides

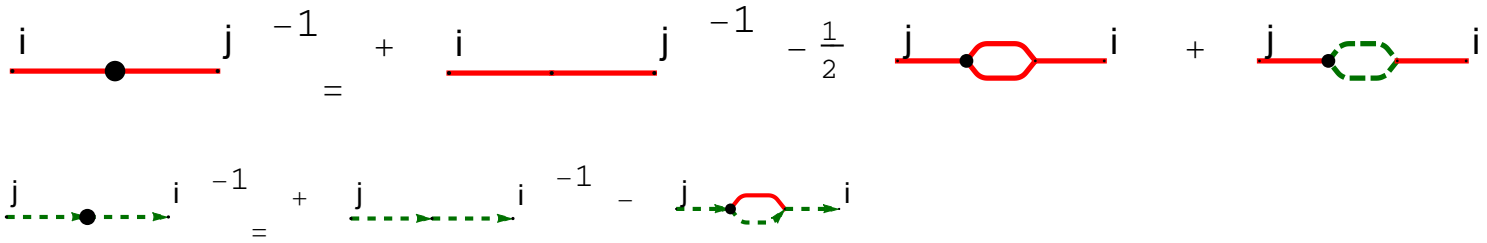
- Perturbation theory
- Symmetries
- Lattice
- Analytic results

Practical obstacle: Manage the system of equations. → Automatization tools

[Alkofer, MQH, Schwenzer '08; Braun, MQH '11; MQH, Mitter '11; `http://tinyurl.com/dofun2`; `http://tinyurl.com/crasydse`]

Truncation of Yang-Mills system

Coupled system of propagators with models for three-point functions:



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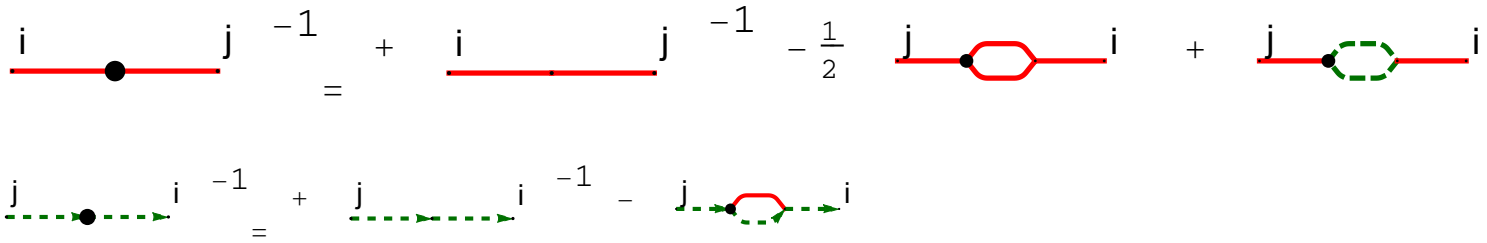
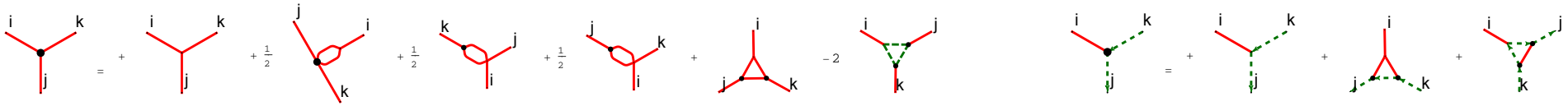


Include three-point functions dynamically [Blum, MQH, Mitter, von Smekal '14]:



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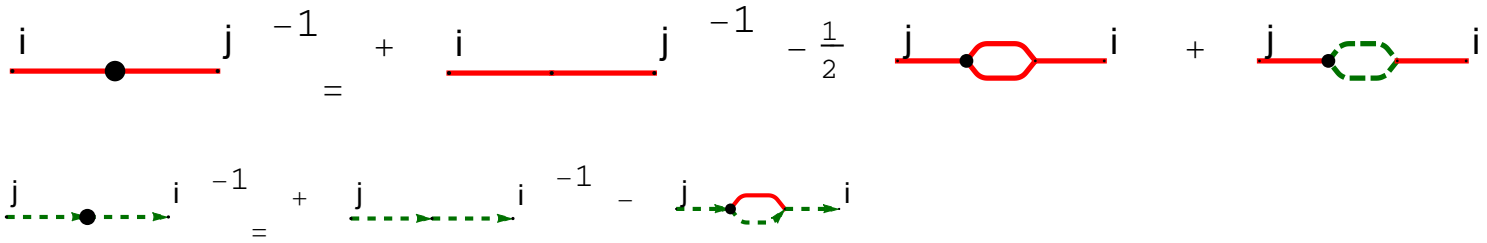

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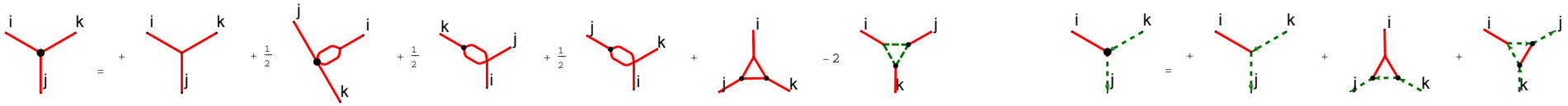

Open questions:

- Four-gluon vertex (in this truncation scheme no dependence on higher n-point functions, see also [Cyrol, MQH, von Smekal '14])
- Two-loop diagrams in propagators (ok for three-point functions)

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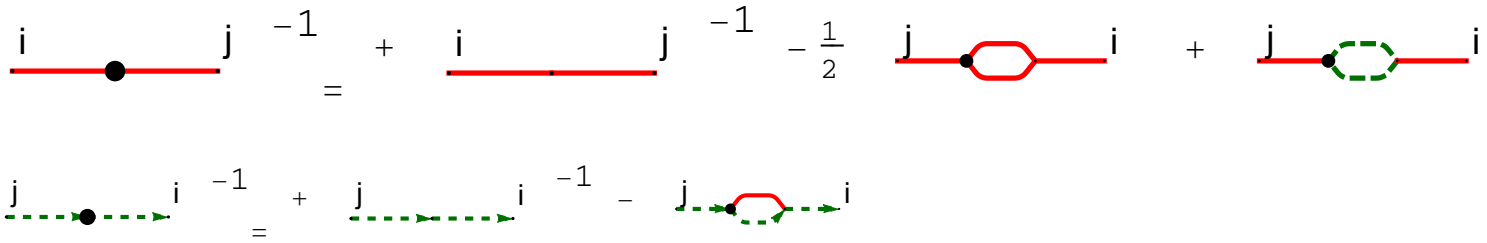

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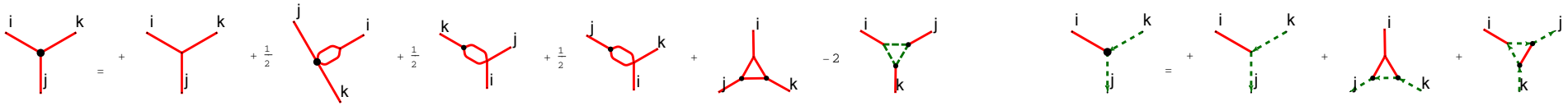

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- Four-gluon vertex (in this truncation scheme no dependence on higher n-point functions, see also [Cyrol, MQH, von Smekal '14])
- Two-loop diagrams in propagators (ok for three-point functions)
- Technical questions: spurious divergences in gluon propagator, RG resummation

Yang-Mills theory in 3 dimensions: Propagator results

$$d = 3$$

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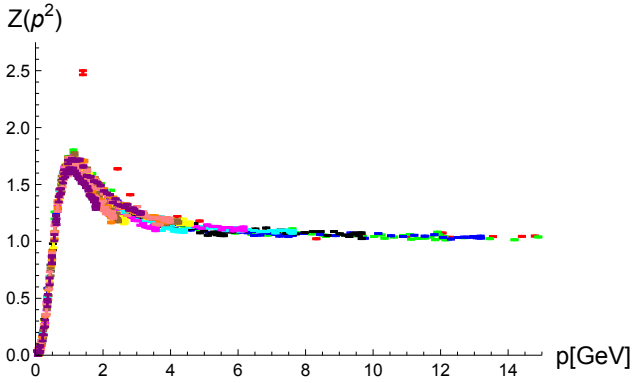
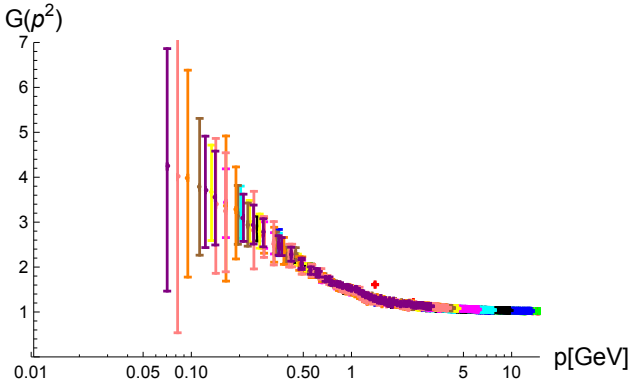
Historically interesting because cheaper on the lattice → easier to reach the IR, e.g., [Cucchieri '99; Cucchieri, Mendes, Taurines '03; Maas '08, '14; Cucchieri, Dudal, Mendes, Vandersickel '11; Bornyakov, Mitrjushkin, Rogalyov '11, '13]





[Maas '14]

Continuum results:

- Coupled propagator DSEs: Maas, Wambach, Grüter, Alkofer '04
- (R)GZ: Dudal, Gracey, Sorella, Vandersickel, Verschelde '08
- YM + mass term: Tissier, Wschebor '10, '11
- DSEs of PT-BFM: Aguilar, Binosi, Papavassiliou '10

Yang-Mills theory in 3 dimensions: Why again?

NB: Numerically not cheaper for functional equations.

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Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle

$\Rightarrow$ Many complications from $d = 4$ absent!

Subtraction of divergences of gluon propagator

1. Logarithmic divergences handled by subtraction at p_0.
2. Quadratic divergences subtracted, coefficient C_{sub}.

$$Z(p^2)^{-1} := \underbrace{Z_\Lambda(p^2)^{-1}}_{\text{calculated right-hand side}} - C_{\mathrm{sub}} \left(\frac{1}{p^2} - \frac{1}{p_0^2} \right)$$

C_{sub} can be calculated anlytically,
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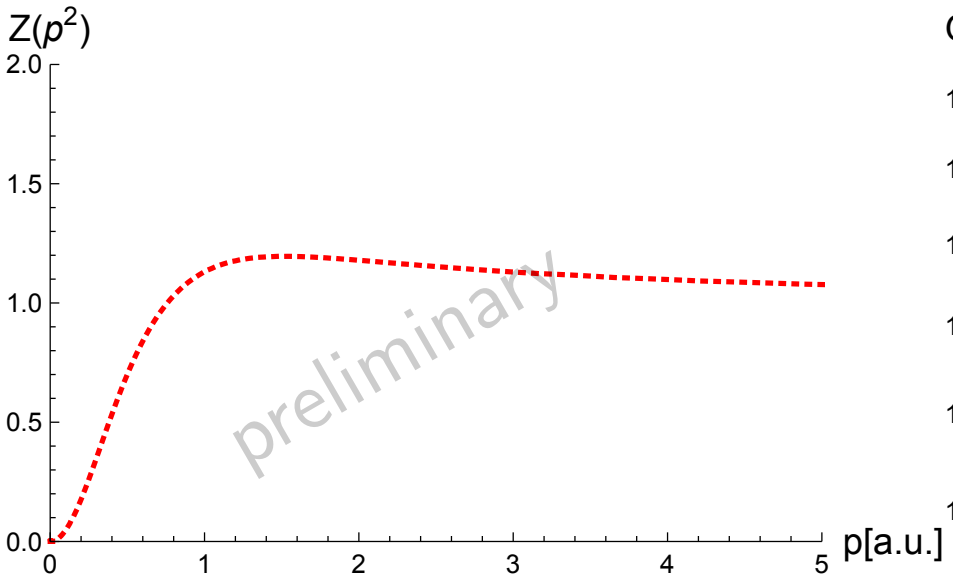
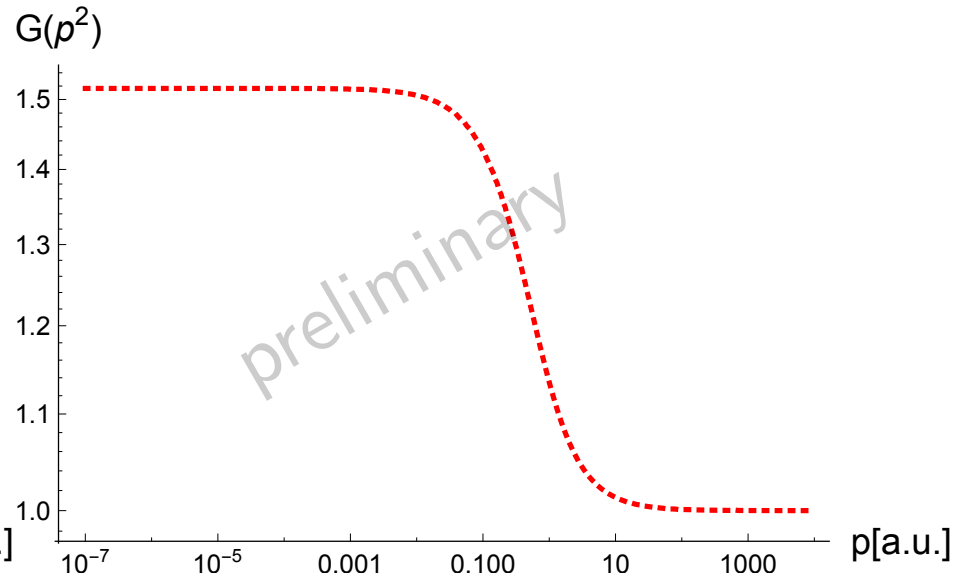
A solution for the propagators

'Standard' truncation of propagators:
1-loop, bare ghost-gluon vertex, three-gluon vertex model





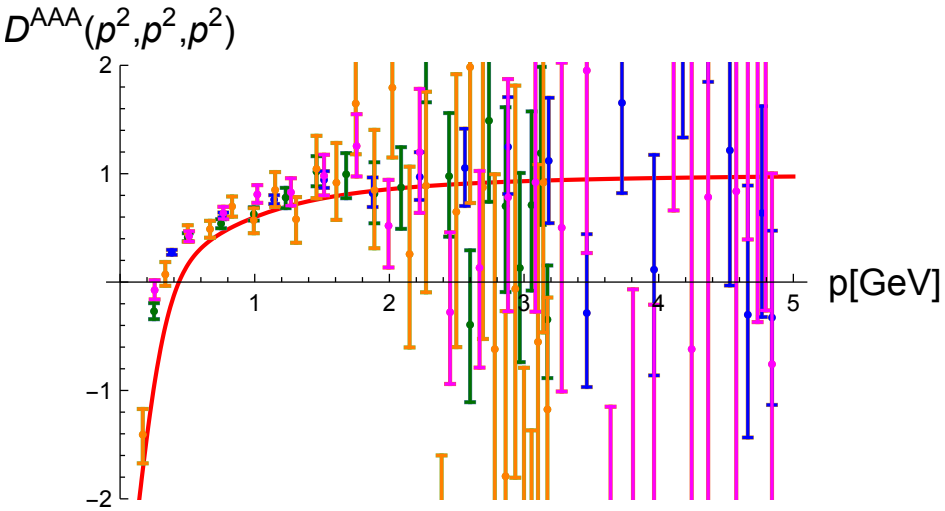
Form of spurious divergences (analytic):

$$C_{sub} = a\,\Lambda + b\,\ln\Lambda$$

Three-gluon vertex model

$$D^{A^3}(x,y,z) = \frac{\overline{p}^2}{\overline{p}^2 + L^2} - G(\overline{p}^2)^3 \frac{L^6}{(L^2+x)(L^2+y)(L^2+z)}$$

$$\overline{p}^2 = \frac{x+y+z}{2}$$

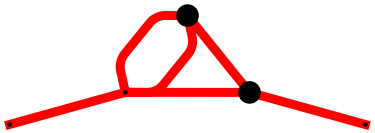


Lattice: [Cucchieri, Maas, Mendes '08]

Not possible to raise the gluon bump further by playing with the vertex models!

Two-loop diagrams

Squint:



Sunset:

4d: [Bloch '03; Mader, Alkofer '12; Meyers, Swanson '14]

Main obstacle: spurious divergences

Spurious divergences

Leading order corrections to subtraction coefficient: $g^4 \to \log(\Lambda)$

Determined by a fit:

- Very small.
- Still large effect.

Two-loop diagrams

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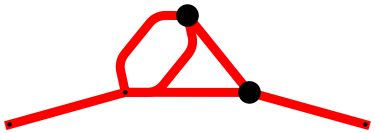

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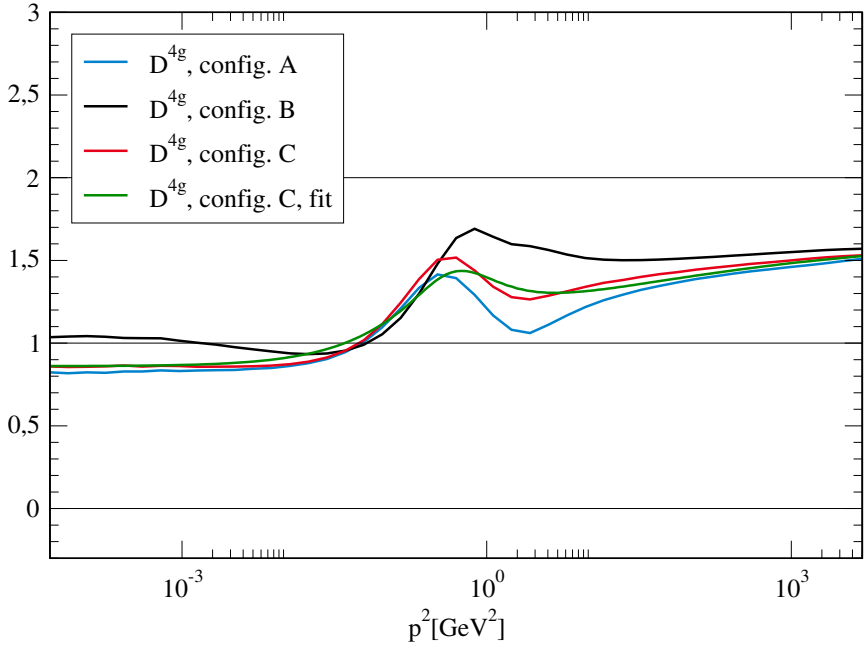
New vertex enters: Four-gluon vertex

Four-gluon vertex

4d: Solution of four-gluon DSE
(full momentum dependence)



[Cyrol, MQH, von Smekal '14]

Similar results by

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Four-gluon vertex

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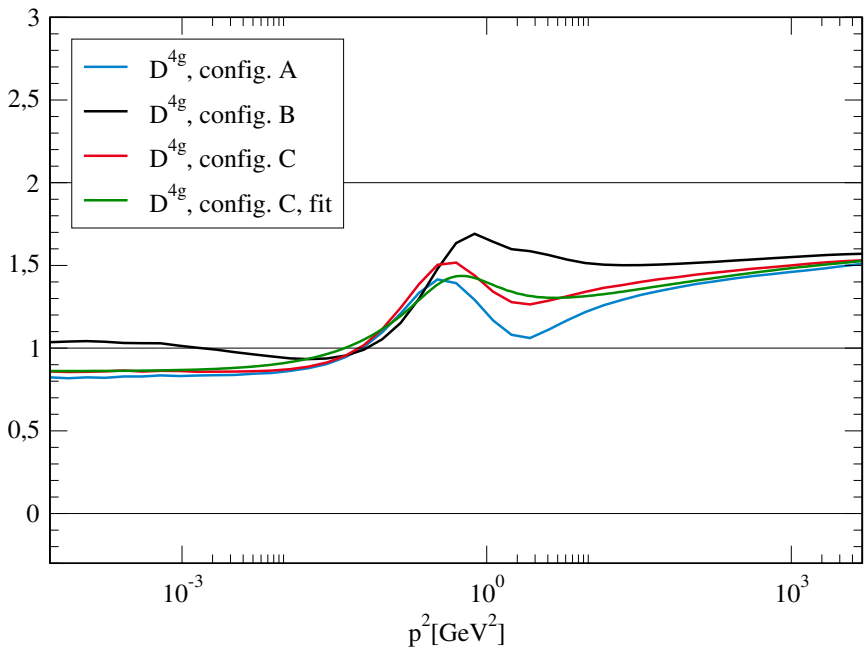

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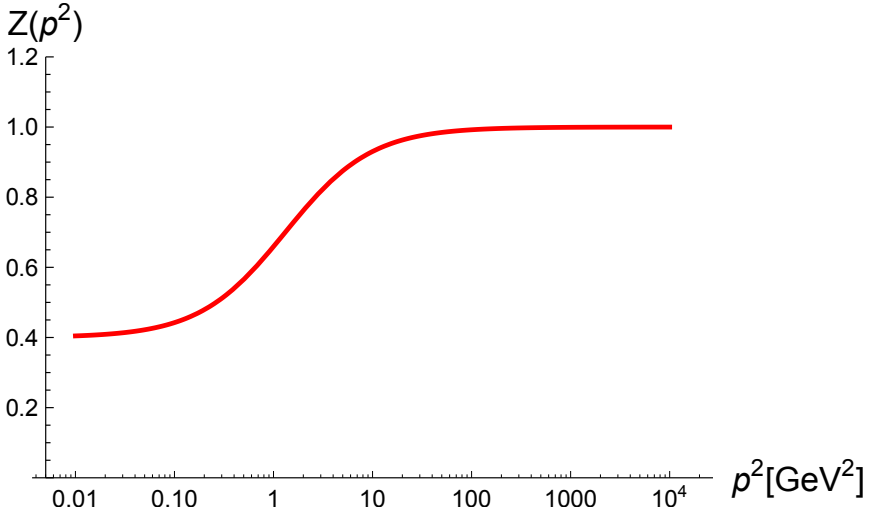
3d:

Simple ansatz with suppression in mid-momentum regime:

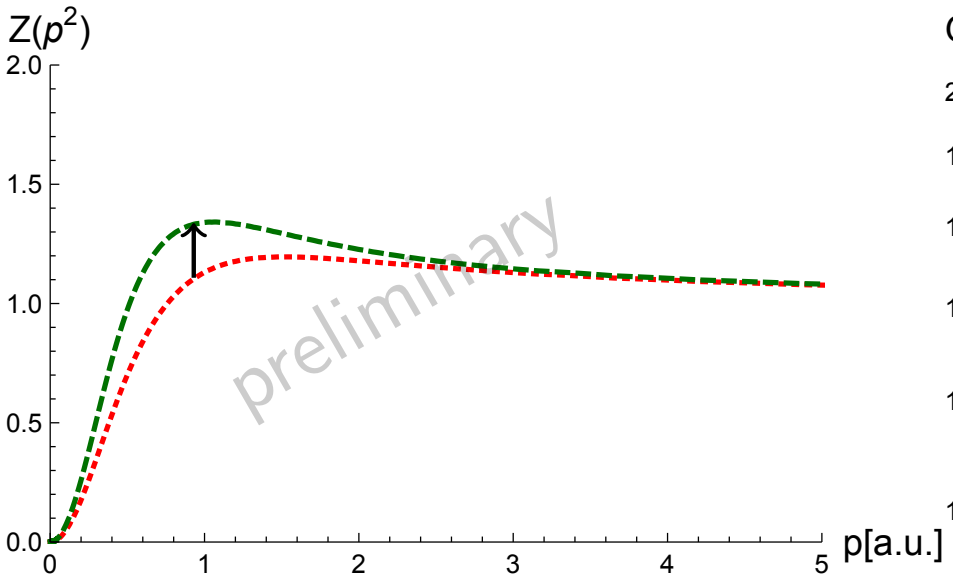
$$D^{AAAA}(x) = \frac{a\,b}{x+b} + 1$$

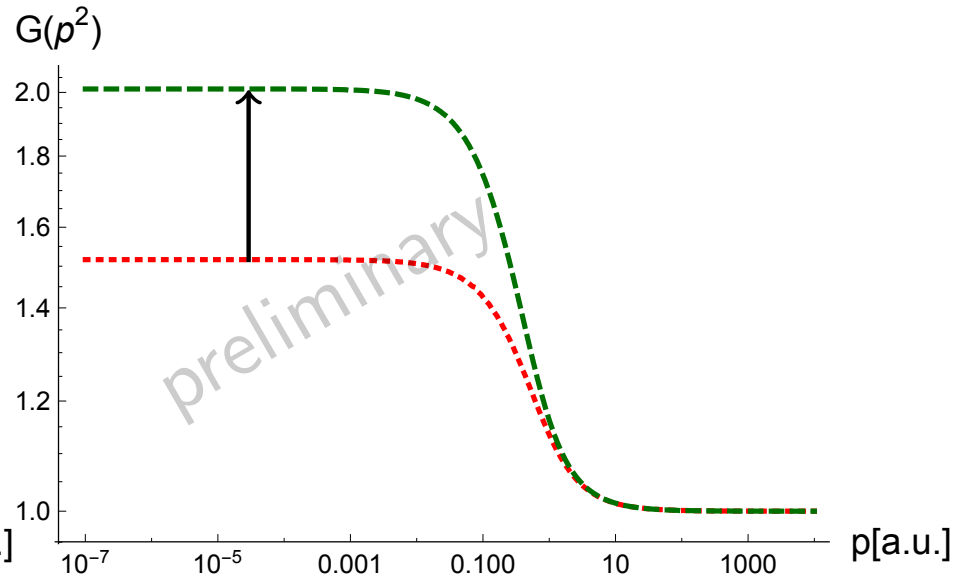


$a = -0.6,\ b = 1.31\ GeV^2$

A solution for the propagators with two-loop diagrams

Improved truncation of propagators:
1- and 2-loops, bare ghost-gluon vertex, three-gluon vertex model





→ Two-loop diagrams essential to allow raising the gluon bump.

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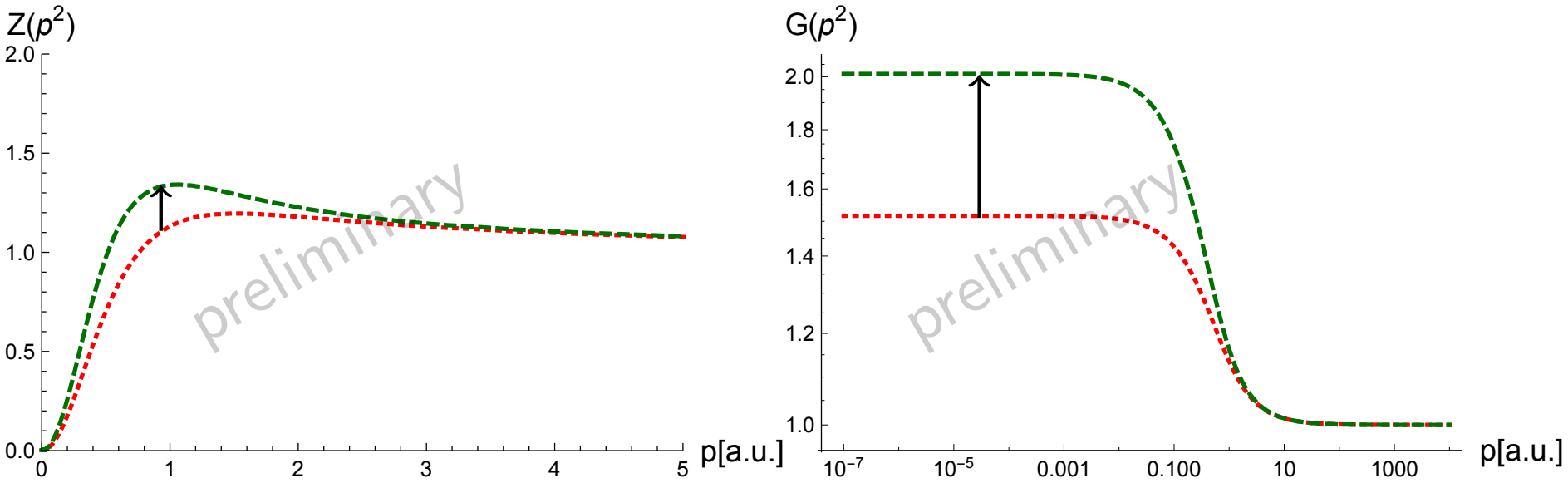
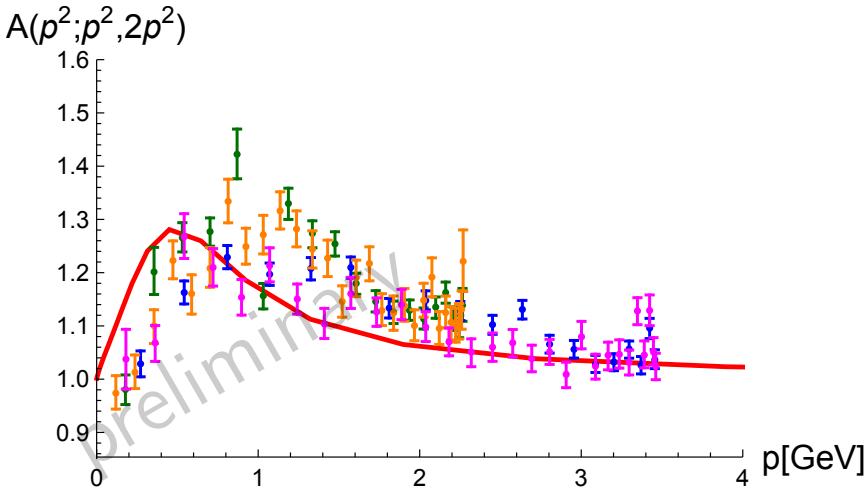
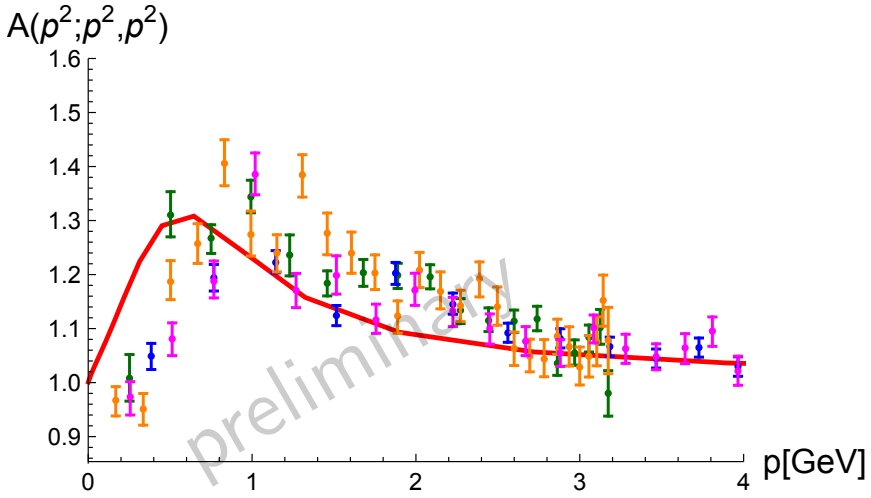


→ Two-loop diagrams essential to allow raising the gluon bump.

⇒ Next step: include vertices dynamically.

Ghost-gluon vertex

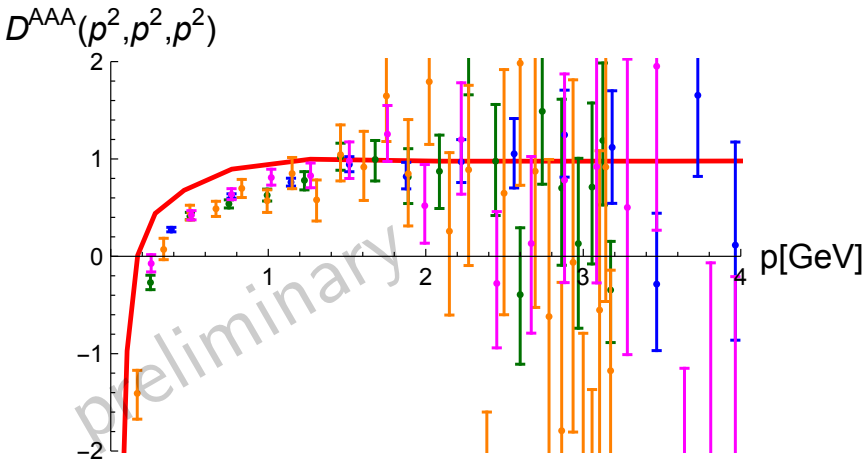


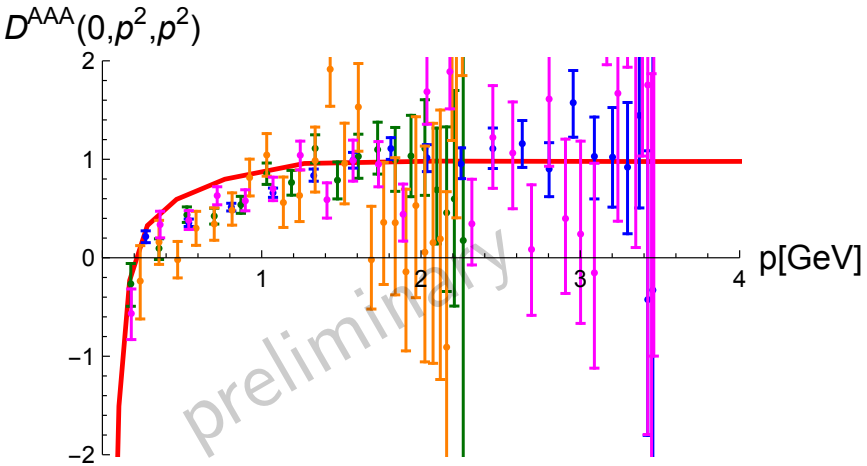


Lattice: [Cucchieri, Maas, Mendes '08]

Full momentum dependence calculated!

Three-gluon vertex





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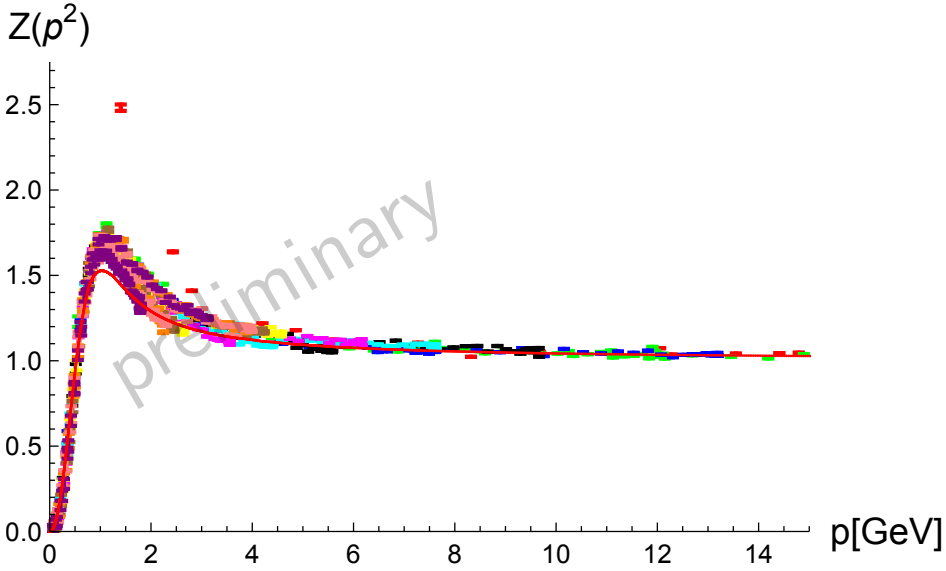
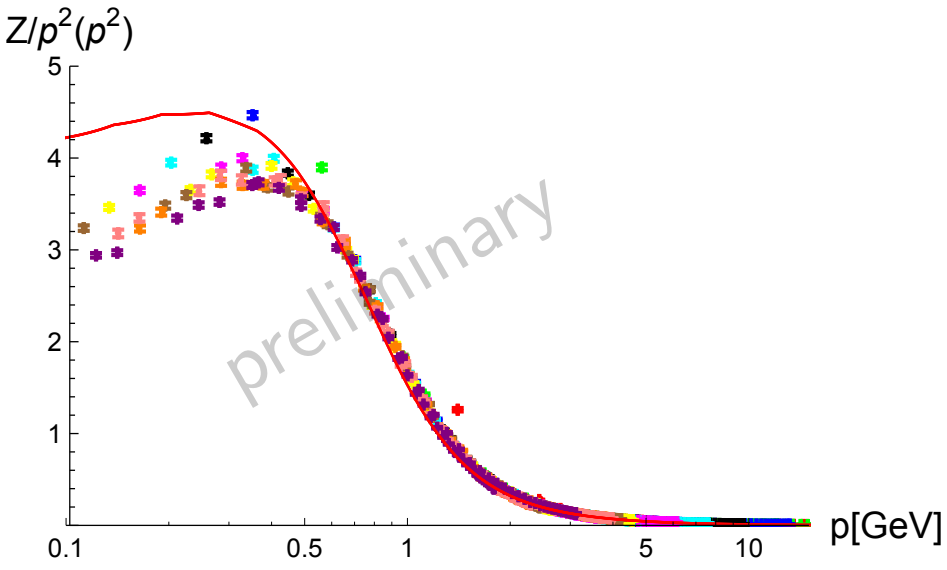
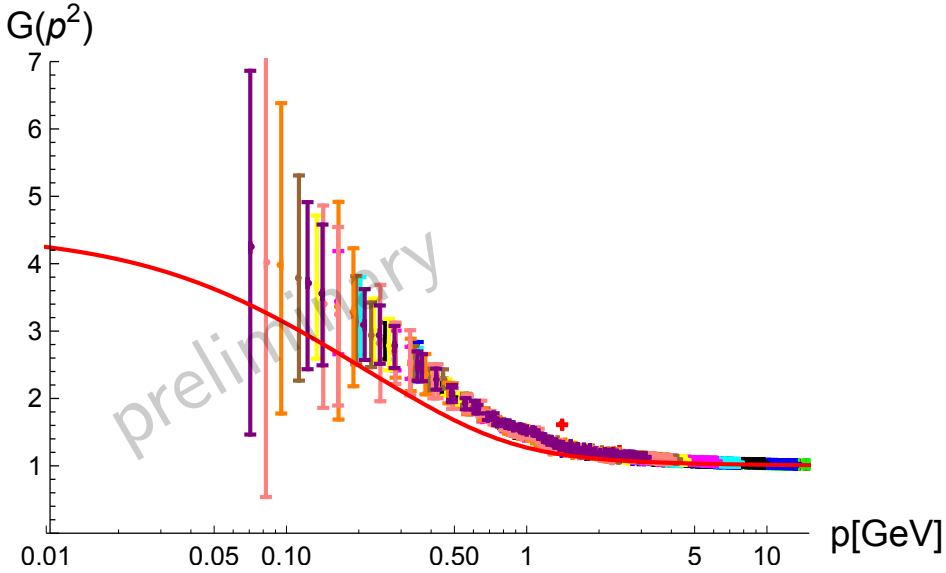
Full momentum dependence calculated!

Note: IR divergent!
Cf. [Peláez, Tissier, Wschebor '13]

Propagators



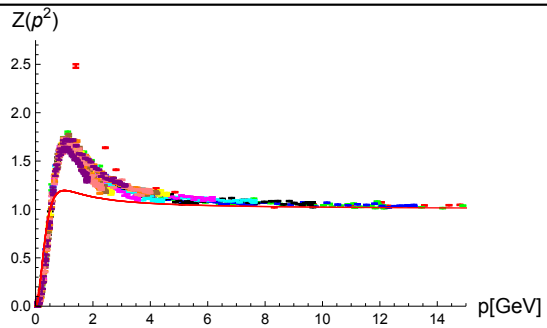
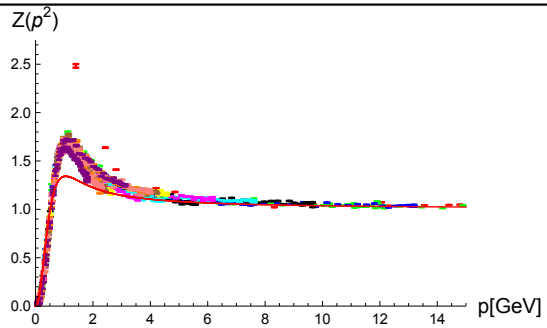
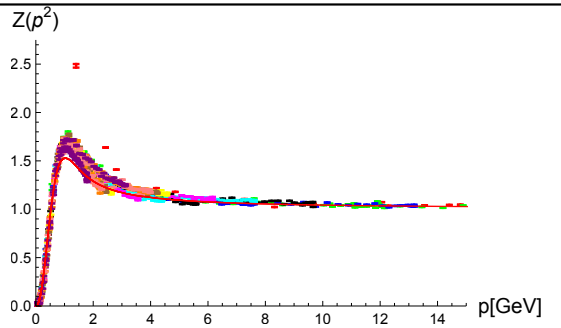




Lattice: [Maas '14]

- Quantitative improvement!
- Small gaps left.

Overview $d = 3$

2-point one-loop	2-point two-loop	3-point	4-gluon	est. max. deviation	
✓	–	model	–	38%	
✓	✓	model	model	23%	
✓	✓	✓	model	8%	

Summary and conclusions

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- Various improvements (two-loop diagrams, dynamic three-point functions) lead to results close to lattice results.
- Missing piece: four-gluon vertex

Implications

- An example of a self-consistent, self-contained truncation of a set of DSEs with quantitative results???

Parameter-less truncation possible!?

- What about 4d?

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