Transport Coefficients and Thermalisation in Classical Field Theories

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Viscosity of hadronic matter

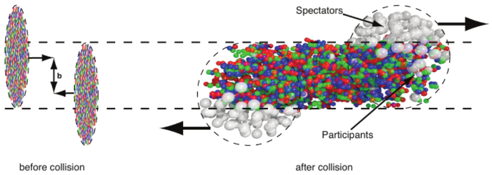


Figure: Heavy-ion collisions

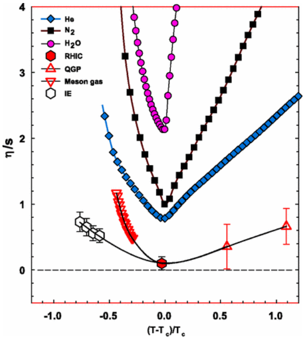


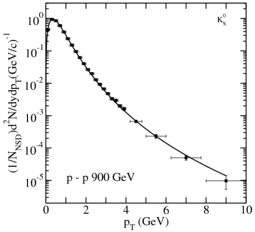
Figure: η/s [1]

- observation: nearly ideal liquid
- relevant quantity: η/s (damping of hydrodynamic waves)

Tsallis distribution in heavy-ion collisions and hadronization

Urmossy, K. and Barnaföldi, G.G. and Harangozó, Sz. and Biró, T.S. and Xu, Z.,
A 'soft + hard' model for heavy-ion collisions, arXiv:1501.02352 [hep-ph], 2015

Cleymans, J. and Worku, D.,
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s}$ = 0.9 TeV at the LHC,
J.Phys.G39:025006, 2012



Khachatryan, Vardan and others, *Transverse-momentum and pseudorapidity distributions of charged hadrons in pp collisions at $\sqrt{s} = 7$ TeV*,
CMS Collaboration, Phys.Rev.Lett. 105.022002, 2010

Aamodt, K. and others,
Production of pions, kaons and protons in pp collisions at $\sqrt{s} = 900$ GeV with ALICE at the LHC, Eur.Phys.J.C71:1655, 2011

Objectives

- thermalization properties
- viscosity (η) in classical field theory
- small transport coefficient, non-trivial distributions $\Leftrightarrow$ strongly interacting system $\rightarrow$ non-perturbative
- Boltzmann equation, MC (less sensitive for $\omega \rightarrow 0$)
- different approach:
- test: classical Φ^4 theory

$$\mathcal{H} = \frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\Phi)^2 + \frac{m^2}{2}\Phi^2 + \frac{\lambda}{24}\Phi^4 \qquad (1)$$

- "toy model" which may show interesting effects mentioned above

Expectation values of local quantities

- Local quantity: $A(\Phi, \Pi)$
- Real measurement is the time average:

$$\langle A(\Phi, \Pi) \rangle = \frac{1}{t} \int_{t_0}^{t_0+t} dt' A(\Phi(t), \Pi(t)) \tag{2}$$

- Inserting δ integral $\langle A(\Phi, \Pi) \rangle =$

$$\int \mathcal{D}\bar{\Phi}\mathcal{D}\bar{\Pi} A(\bar{\Phi}, \bar{\Pi}) \frac{1}{t} \int_{t_0}^{t_0+t} dt' \delta(\bar{\Phi} - \Phi(t))\delta(\bar{\Pi} - \Pi(t)) \tag{3}$$

- the second part is a histogram $f(\bar{\Phi}, \bar{\Pi})$
- Time-average $\rightarrow$ Ensemble average

$$\langle A(\Phi, \Pi) \rangle = \int \mathcal{D}\bar{\Phi}\mathcal{D}\bar{\Pi} A(\bar{\Phi}, \bar{\Pi}) f(\bar{\Phi}, \bar{\Pi}) \tag{4}$$

- e.g. canonical: $e^{-\beta\mathcal{H}}$

Simulation

- canonical equations, periodic boundary conditions, leap-frog algorithm
- initial conditions: $\left\{\Pi\left(t_0 + \frac{\delta t}{2}\right), \Phi(t_0)\right\}$
- uniform and $f(\Pi) \sim \mathrm{sech}\left(\frac{\pi}{2}\Pi\right) \rightarrow$ hyperbolic secant distribution
- Canonical eq. of $\dot{\Phi}$ (1st part of time step): Initial condition $\rightarrow \Phi(t_0 + \delta t)$
- Canonical eq. of $\dot{\Pi}$ (2nd part of time step): $\left\{\Phi(t_0 + \delta t), \Pi\left(t_0 + \frac{\delta t}{2}\right)\right\} \rightarrow \Pi\left(t_0 + \frac{3\delta t}{2}\right)$
- input parameters: N^3 lattice size, $a = 1$ (grid), λ (interaction), m^2 Lagrangian-mass

Total energy

$$E = \sum_{\mathbf{i}\in U} \frac{1}{2}\Pi_{\mathbf{i}}^2 + \frac{1}{2}(\nabla\Phi)_{\mathbf{i}}^2 + \frac{m^2}{2}\Phi_{\mathbf{i}}^2 + \frac{\lambda}{24}\Phi_{\mathbf{i}}^4, \qquad (5)$$

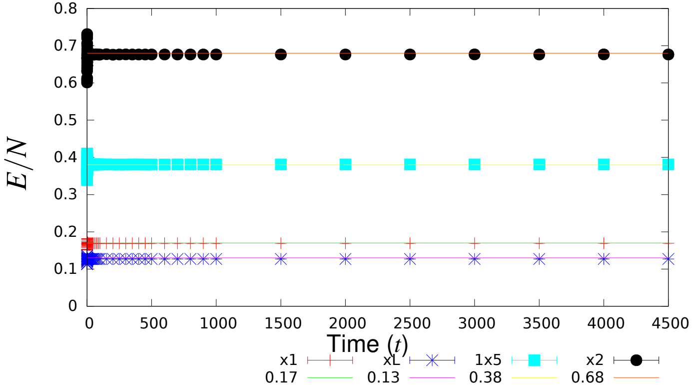


Figure: Time dependence of E/N where $N = 50^3$

Temperature $\langle|\Pi_k|^2\rangle = 2N^3T$

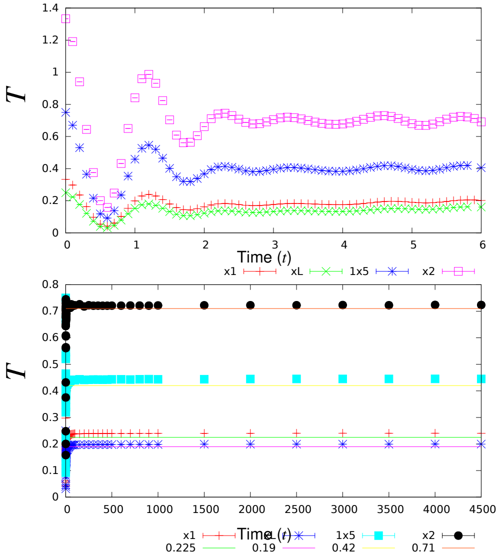
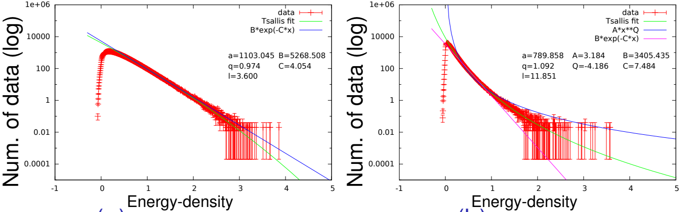


Figure: Time dependence of temperature

Energy histogram

Early time energy-distribution function is not Boltzmannian



(a) uniform init. (b) sech init.

Figure: Different fits on logscale energy histogram

Tsallis distribution is an excellent fit!

$$f(x) = a\left[1 + (q-1)\beta x\right]^{\frac{1}{1-q}} \tag{6}$$

$\rightarrow$ consider the time evolution of q

Not Tsallis?

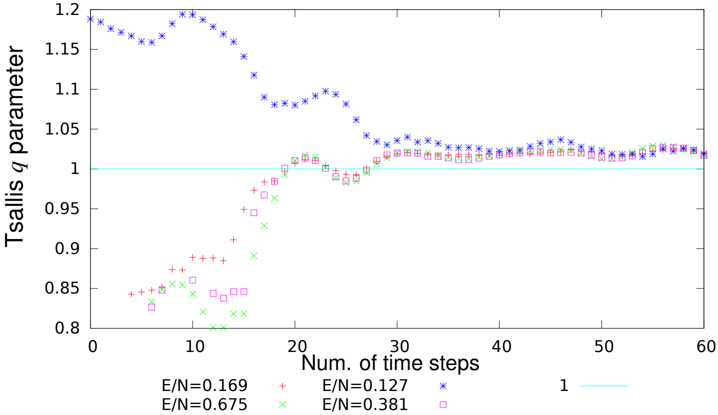


Figure: Time dependence of the Tsallis parameter

Tsallis?

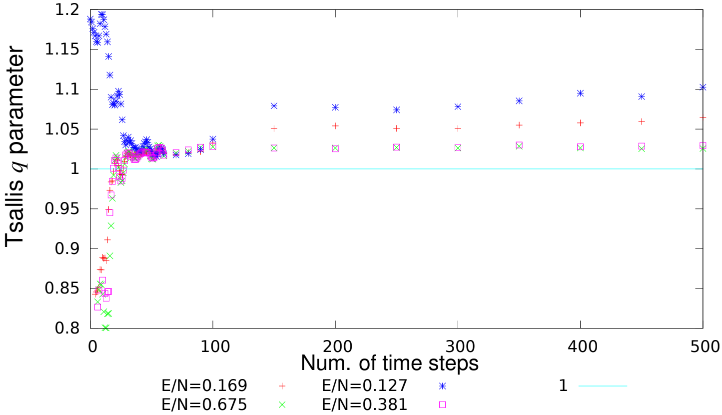


Figure: Time dependence of the Tsallis parameter

Just pre-thermal?

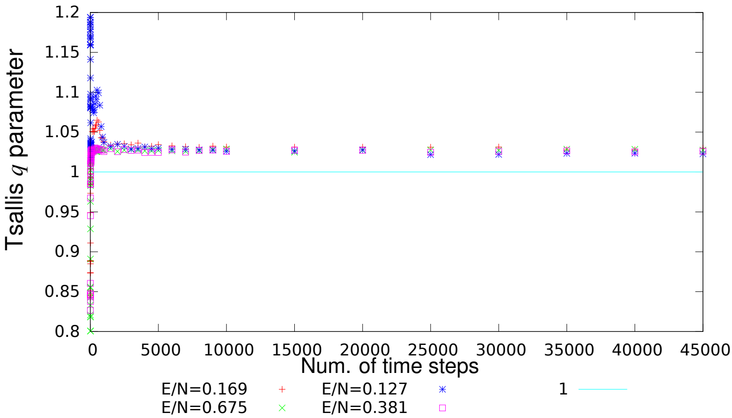


Figure: Time dependence of the Tsallis parameter

$$q \approx 1.026$$

$\Pi(x)$ histogram

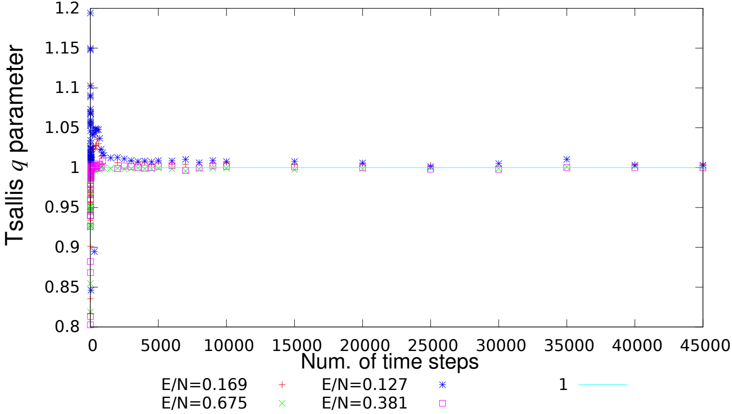
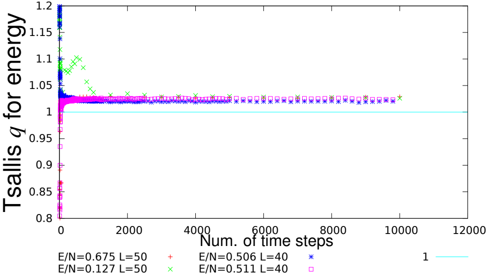
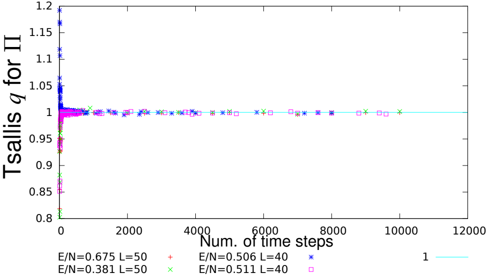


Figure: Time dependence of the Tsallis parameter for Π histogram

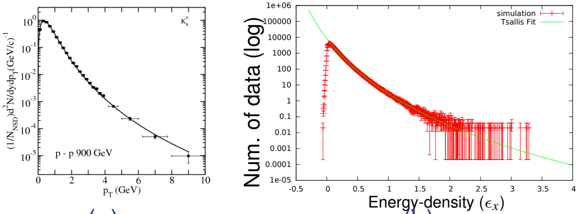
$$q \approx 0.999$$

Lattice size





Interpretation



(a) (b)

Figure: Experiment vs. Φ^4 simulation

- suggestions:
- hadrons are created locally
- creation probability depends on local energy density
- local energy density distribution can be determined by computer simulations $\rightarrow$
- it might be a tool for measuring hadron distribution function

Bibliography

Roy A. Lacey, N. N. Ajitanand, J. M. Alexander, P. Chung, W. G. Holzmann, M. Issah, A. Taranenko, P. Danielewicz, and Horst Stöcker.
Has the qcd critical point been signaled by observations at the bnl relativistic heavy ion collider?
Phys. Rev. Lett., 98:092301, Mar 2007.

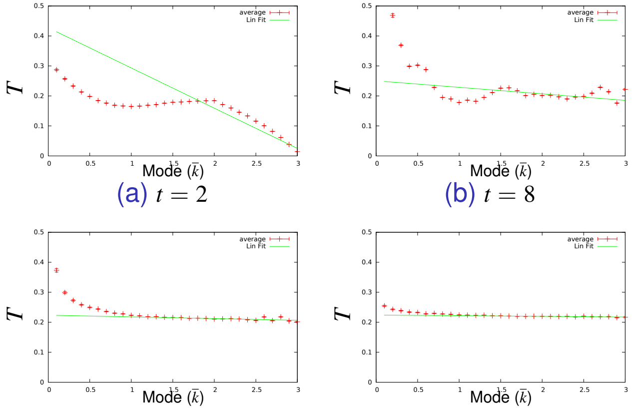
A. Jakovac.
Viscosity of the scalar fields from the classical theory.
Phys.Lett., B446:203–208, 1999.

En-ke Wang, Ulrich W. Heinz, and Xiao-fei Zhang.
Spectral functions for composite fields and viscosity in hot scalar field theory.
Phys.Rev., D53:5978–5981, 1996.

Thank you for your attention!

Equipartition

equilibrium $\rightarrow$ equipartition



(a) $t = 2$ (b) $t = 8$

(c) $t = 200.1$ (d) $t = 4500.1$

Figure: Equipartition during time evolution ($\lambda = 5$)