# Islands of stability – Breathers in asymptotically anti-de Sitter space-times

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# Outline

1. Motivations
2. anti deSitter (AdS) spacetimes
3. Instability of asymptotically AdS
4. scalar AdS breathers – perturbatively
5. numerical results – stability islands in AdS

# Motivation(s)

Stability of vacuum space-times

Minkowski, DeSitter/anti deSitter space-times
correspond to "ground states" of gravity,

maximally symmetric solutions of vacuum Einstein's eqs.
with zero, positive/negative cosmological constant .

Minkowski, dS/AdS spacetimes are all linearly stable

"non-linear" stability? (small perturbations remain small).

Stability of both Minkowski and of dS have been shown
(Christodoulou, Klainerman; Friedrich)

Strong evidence for instability of AdS (Anderson; Dafermos,
Holzegel)

generic perturbations become large and form black holes
(Bizon,Rostworowski)

# Motivation(s)

AdS/CFT correspondence relates

$d+1$ dim. gravity in asymptotically AdS

to a $d$ dim. QFT defined on the boundary of AdS

Black hole formation $\Leftrightarrow$ thermalization in QFT

Black holes $\Leftrightarrow$ thermal states

$\rightarrow$ transport properties in strongly coupled QFTs

$\rightarrow$ holographic description of condensed matter systems

AdS instability conjecture:
black holes form from arbitrarily small, generic perturbations in asymptotically AdS

strong evidence for time-periodic objects (breathers)
if stable $\rightarrow$ no black hole formation
$\rightarrow$ no thermalization on the CFT side

# 1+d dimensional anti de Sitter space-time

AdS is the maximally symmetric solution of Einstein's eqs. with a negative cosmological constant, $\Lambda$:

$$ds^2 = -(1+k^2r^2)dt^2 + \frac{dr^2}{1+k^2r^2} + r^2 d\Omega_{d-1}^2$$

in Schwarzschild coordinates, with $\Lambda = -\frac{1}{2}d(d-1)k^2$.

Absolute value of the outwards acceleration of constant $r$ observers:

$$a = \frac{k^2 r}{\sqrt{1+k^2r^2}} \xrightarrow[r\to\infty]{} k$$

AdS background corresponds to an effective attractive force

$g_{rr} = (1+k^2r^2)^{-1} \xrightarrow[r\to\infty]{} 0$, $r \to \infty$ corresponds to what?

# 1+d dimensional anti de Sitter space-time

Introduce a radial distance coordinate, $r = \sinh(k\rho)/k$

$$ds^2 = -\cosh^2(k\rho)dt^2 + d\rho^2 + \frac{1}{k^2}\sinh^2(k\rho)d\Omega^2_{d-1}$$

i.e. $r \to \infty$ does correspond to spatial infinity. A conformal coordinate system is obtained by setting

$$r = \frac{1}{k}\tan x \quad 0 \leq x \leq \frac{\pi}{2}\,; \qquad\qquad t = \frac{1}{k}\tau\,,$$

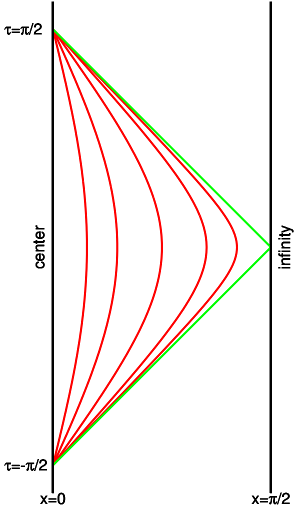
when AdS takes the form:

$$ds^2 = \frac{1}{k^2\cos^2 x}\left(-d\tau^2 + dx^2 + \sin^2 x\, d\Omega^2_{d-1}\right)$$

conformal to a region of the Einstein static universe

– null geodesics are $45^\circ$ lines

# 1+d dimensional anti de Sitter space-time




Anti-de Sitter spacetime

$$ds^2 = \frac{1}{k^2 \cos^2 x} \left(-d\tau^2 + dx^2 + \sin^2 x \, d\Omega^2\right)$$

boundary at timelike infinity ⇒ boundary conditions: reflecting b.c.
light rays travel to infinity and back in finite time

essential difference from asymptotically Minkowski or deSitter cases

ingredient to instability of AdS – a wave packet can bounce back many times to the center, and finally collapse to a black hole

# AdS breathers

$d+1$ dimensional Einstein's equations coupled to a massless real scalar field:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad , \quad T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}$$

together with the wave equation

$$\nabla^{\mu}\nabla_{\mu}\phi = 0$$

look for spherically symmetric solutions:

$$\mathrm{d}s^2 = \frac{L^2}{\cos^2 x}\left(-Ae^{-2\delta}\mathrm{d}t^{\,2} + \frac{1}{A}\mathrm{d}x^2 + \sin^2 x\,\mathrm{d}\Omega^2_{d-1}\right)$$

where $A = A(t,x)$ and $\delta = \delta(t,x)$; $L^2 = -d(d-1)/2/\Lambda$

– anti-de Sitter space-time corresponds to $A = 1$ and $\delta = 0$

# AdS breathers: main results

G.Fodor, P. Forgács and P. Grandclément,
Phys. Rev. D **92**, 025036 (2015), arXiv:1503.07746 [gr-qc]

We apply three methods:

- Spectral code for constructing time-periodic solutions
- Time-evolution code to study stability
- Perturbation theory: small-amplitude expansion $\Rightarrow$ analytical results

Extension of the results of M. Maliborski and A. Rostworowski, *Phys. Rev. Lett.* **111,** 051102 (2013)

– methods that work well for $2n+1$ spacetime dimensions
– results presented for $4+1$ dimensions

We find stability up to a maximal mass,

higher amplitude breathers exist, but they are unstable
also find some strange resonance-like behaviour

# Small-amplitude expansion

The scalar field and the metric functions are expanded in powers of a small parameter $\varepsilon$

$$\phi = \sum_{n=0}^{\infty} \phi^{(2n+1)} \varepsilon^{2n+1} \ , \quad A = 1 + \sum_{n=1}^{\infty} A^{(2n)} \varepsilon^{2n} \ , \quad \delta = \sum_{n=1}^{\infty} \delta^{(2n)} \varepsilon^{2n}$$

To first order in $\varepsilon$: the metric is AdS, $\phi^{(1)}(x,t)$ is given as:

$$\phi^{(1)}(x,t) = p_m(x) \cos(\omega_m t) \quad , \qquad m \geq 0 \text{ integer}$$

$p_m(x)$ can be given with Jacobi polynomials, and the allowed frequencies are

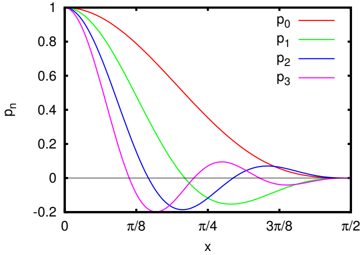
$$\omega_m = d + 2m \,,$$

$\phi^{(1)}(x,t)$ is spatially localized, time-periodic $\rightarrow$ breather.

# Small-amplitude expansion

all $\omega_m$ are integers $\Rightarrow$ fully resonant spectrum
$\rightsquigarrow$ turbulent instability $\rightsquigarrow$ black hole formation

For $d = 3$ the functions $p_m$ are given as:




$$p_0 = \cos^3 x$$

$$p_1 = \frac{\cos^3 x}{3}\,[4\cos(2x) - 1]$$

$$p_2 = \frac{\cos^3 x}{3}\,[3\cos(4x) - 2\cos(2x) + 2]$$

$$p_3 = \frac{\cos^3 x}{15}\,[12\cos(6x) - 9\cos(4x) + 18\cos(2x) - 5]$$

$m$ gives the number of nodes.

# Small-amplitude expansion

general solution to 1st order is a superposition:

$$\phi^{(1)} = \sum_{m=0}^{\infty} a_m \cos(\omega_m t + b_m) p_m \quad , \quad \omega_n = d + 2m$$

with arbitrary amplitudes, $a_m$ and phases $b_m$.
To $\varepsilon^3$ order, however, in $\phi^{(3)}$ secular terms appear if more then one $a_m$ is nonzero

Seems impossible to remove all of the secular terms in higher orders, except if one starts with a single mode.

There is a one-parameter family of solutions emerging from each $p_n$ linearized mode

# Numerical construction of AdS breathers

investigate the family emerging from the nodeless solution

Initial guess for numerical iteration: linearized solution $p_0 \cos(3t)$

KADATH library developed by Philippe Grandclément at Observatoire de Paris - Meudon

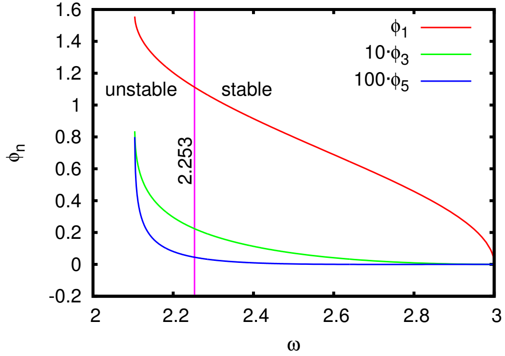
– multidomain spectral method

– radial direction: Chebyshev polynomials

– time direction: Fourier decomposition

$$\phi = \sum_{\substack{k=1 \\ odd}}^{\infty} \phi_k \cos(k\omega t) \ , \quad A = \sum_{\substack{k=0 \\ even}}^{\infty} A_k \cos(k\omega t) \ , \quad \delta = \sum_{\substack{k=0 \\ even}}^{\infty} \delta_k \cos(k\omega t)$$

## Central value of the Fourier modes as function of oscillation frequency

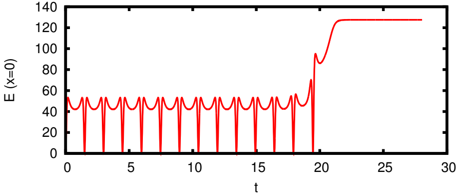



nodeless AdS breathers with frequencies $\omega > 2.253$ are stable

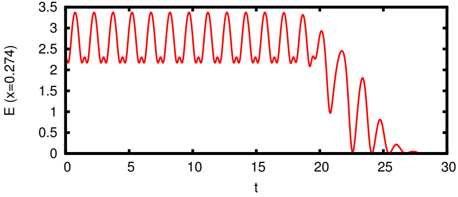
Energy density as function of time for an unstable configuration

– at the center

– at a radius larger than the one where horizon appears

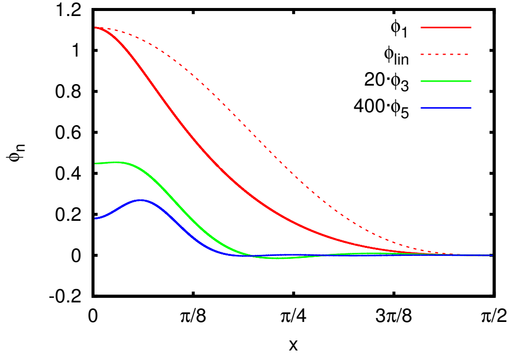



energy density starts to increase, but time coordinate stalls before horizon



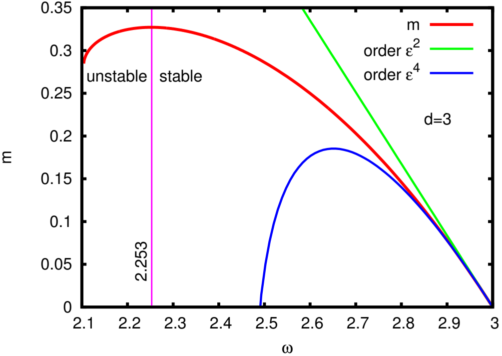

scalar field falls into the black hole

Radial profile for the first three modes of the scalar field for the largest mass stable AdS breather




– more compact than the linear solution, but similar shape

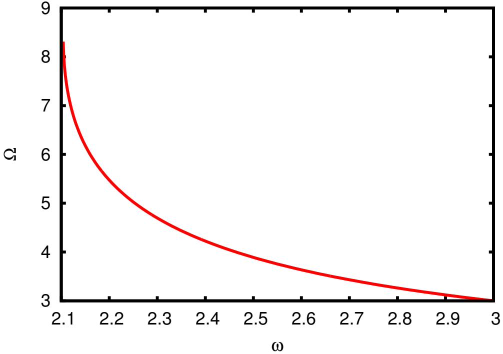
## Mass as function of the oscillation frequency




AdS breather becomes unstable when the total mass starts to decrease with increasing central density

First two orders of the small-amplitude expansion is also plotted – in order to get $\varepsilon^4$ order results one has to calculate to $\varepsilon^6$ order

## Central frequency $\Omega$ as function of asymptotic frequency $\omega$

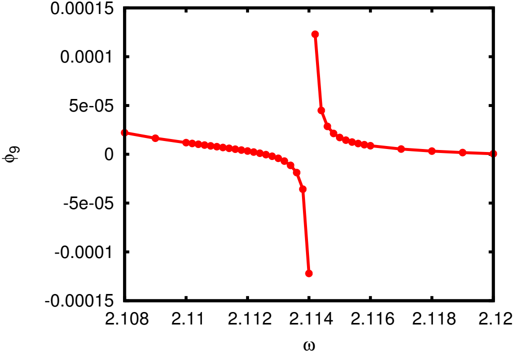



As the amplitude grows

the frequency observed outside the breather decreases

the frequency measured by a central observer grows

## There are narrow resonance-like structures appearing in higher Fourier modes




– the increase is most apparent in one of the modes
– for $d = 4$ a similar peak was found in $\phi_5$
– they are in the unstable domain

# Concluding remarks

Periodic solutions, up to a certain amplitude, are on
"stability islands"
– general configurations collapse into black holes

AdS/CFT correspondence
– periodic solutions correspond to states that never thermalize

There are other asymptotically AdS localized regular configurations

- static axially symmetric electromagnetic states
  Herdeiro and Radu, *Phys. Lett. B* **749**, 393 (2015)
- vacuum gravitational wave geons
  Dias, Horowitz and Santos, *CQG* **29**, 194002 (2012)
  – helical symmetry