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MATHEMATISCH-
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FAKULTÄT

Hamiltonian Approach to QCD in Coulomb gauge

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Outline

1. Motivation
2. Canonical Dyson–Schwinger Equations
   - Hamiltonian approach to QCD
   - Vacuum wave functional and CDSEs
3. Variational solution for the Yang–Mills sector
   - $T = 0$
   - $T \neq 0$
4. Fermions
5. Conclusions

(Hamiltonian Approach to) Coulomb gauge

The good. . .

- well-defined (lattice) Hamiltonian
- analogy to standard quantum mechanics
- appeals to physical intuition

. . . the bad. . .

- non-covariant
- renormalization not clear

. . . and the ugly

- calculations become quickly very involved

Hamilton operator of QCD in Coulomb gauge

Steps

- start from canonically quantized theory in temporal gauge $A_0 = 0$
- eliminate longitudinal degrees of freedom by means of Gauss's law

$$H = \frac{1}{2}\left[-\mathcal{J}_A^{-1}\frac{\delta}{\delta A}\mathcal{J}_A\frac{\delta}{\delta A} + B^2\right] + \psi^\dagger(-\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m)\psi \\ - g\psi^\dagger\boldsymbol{\alpha}\cdot\mathbf{A}\,\psi + \frac{g^2}{2}\mathcal{J}_A^{-1}\,\rho\,\mathcal{J}_A\,F_A\,\rho$$

- B is the non-abelian magnetic field
- $\rho^a = \psi^\dagger t^a\psi - \mathrm{i}f^{abc}A^b\frac{\delta}{\delta A^c}$ is the colour charge density
- $F_A = (-\partial\cdot D)^{-1}(-\partial^2)(-\partial\cdot D)^{-1}$ is the Coulomb kernel

Static Green's functions

V.e.v. of an operator

$$\langle K \rangle = \int \mathcal{D}A\, \mathcal{J}_A\, \mathcal{D}\xi^\dagger\, \mathcal{D}\xi\, \Psi^*[A,\xi,\xi^\dagger]\, K\, \Psi[A,\xi,\xi^\dagger]$$

- $\mathcal{J}_A = \mathrm{Det}(-\partial \cdot D)$ (with $D = \partial + A$) is the Faddeev–Popov determinant of Coulomb gauge
- integration over transverse field configurations
- ξ and $\xi^\dagger$ are Grassmann fields
- Ψ is the vacuum wave functional

The expectation values of products of fields

$$\langle AA \rangle, \quad \langle \xi\xi^\dagger \rangle, \quad \langle \xi\xi^\dagger A \rangle, \ \ldots$$

are the static (equal-time) Green functions.

Vacuum wave functional

Formal equivalence to Lagrangian approach

Writing the vacuum wave functional as

$$|\Psi[A,\xi,\xi^\dagger]|^2 =: \exp\Big\{-S[A,\xi,\xi^\dagger]\Big\}$$

we have an Euclidean QFT defined by an “action” $S[A,\xi,\xi^\dagger]$.

Expansion of the vacuum wave functional

$$S[A,\xi,\xi^\dagger] = \frac{1}{2}\,\gamma_2 A^2 + \frac{1}{3!}\gamma_3 A^3 + \frac{1}{4!}\gamma_4 A^4 + \xi^\dagger(\bar\gamma + \bar\Gamma_0 A)\xi + \dots$$

Kernels of the vacuum wave functional

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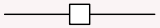
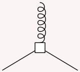
“Bare” vertices?

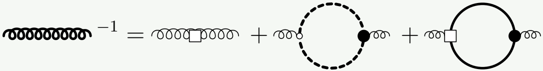
The coefficients in the vacuum wave functional play the role of the bare vertices, but

- are non-local functions
- have a non-trivial expansion in powers of the coupling
- will be represented diagrammatically by small empty boxes

$\gamma_2 =$  , $\bar{\gamma} =$  , $\bar{\Gamma}_0 =$  , $\ldots$

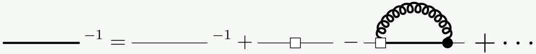
Propagator DSEs

Gluon propagator $\langle AA \rangle \equiv 1/2\Omega(\mathbf{p})$



Ghost propagator $\langle (-\partial \cdot D)^{-1} \rangle$

Quark propagator $\frac{1}{2}\langle [\psi, \psi^\dagger] \rangle$



Not quite equations of motion, rather relations between the Green functions and the kernels of the wave functional.

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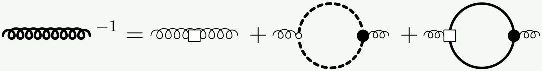

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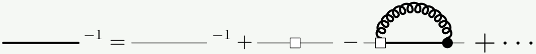

Not quite equations of motion, rather relations between the Green functions and the kernels of the wave functional.

Kernels of the vacuum wave functional

$$S[A,\xi,\xi^\dagger] = \frac{1}{2}\,\gamma_2 A^2 + \frac{1}{3!}\gamma_3 A^3 + \frac{1}{4!}\gamma_4 A^4 + \xi^\dagger(\bar{\gamma} + \bar{\Gamma}_0 A)\xi + \ldots$$

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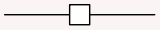
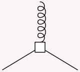
$\gamma_2 =$  , $\bar{\gamma} =$  , $\bar{\Gamma}_0 =$  , $\ldots$

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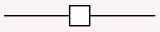
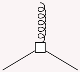
$\gamma_2 =$  , $\bar\gamma =$  , $\bar\Gamma_0 =$  , $\dots$

- are not known... $\Rightarrow$ **variational kernels**

Variational method

- evaluate the energy in the state defined by the chosen Ansatz
- use the CDSEs to express the energy density as a function of the variational kernels
- minimize the energy by taking functional derivatives w.r.t. the variational kernels

$$\Downarrow$$

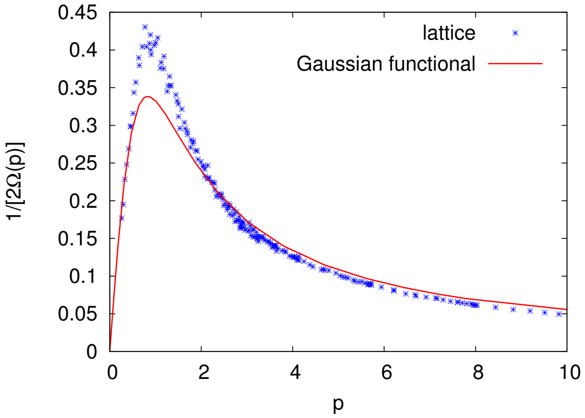
This gives a set of gap equations, which can be combined with the CDSEs.

What we've got

- IR behaviour of propagators
- linearly rising potential between static charges
- Polyakov loop potential and deconfinement phase transition

Yang–Mills sector

Gluon propagator with non-Gaussian functional



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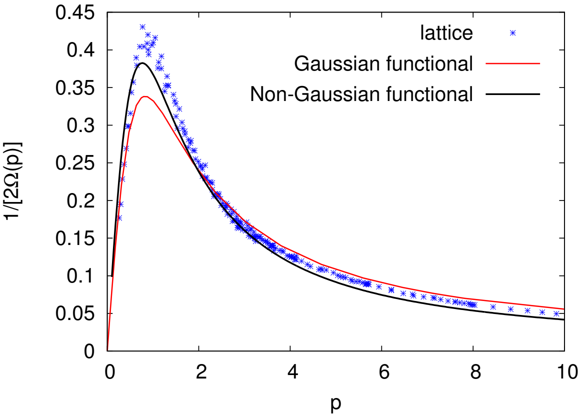
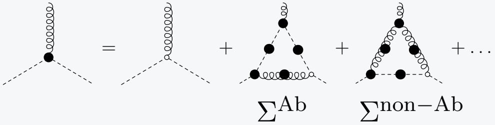
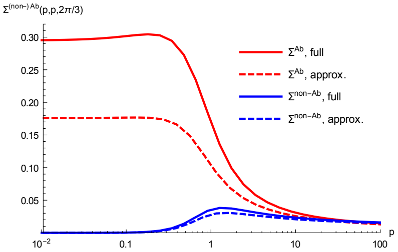


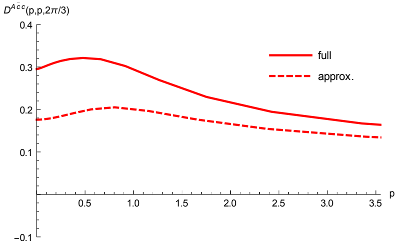
Yang–Mills sector

Ghost-gluon vertex

Truncated CDSE







Pictures by M. Huber

Different approaches to non-vanishing temperature

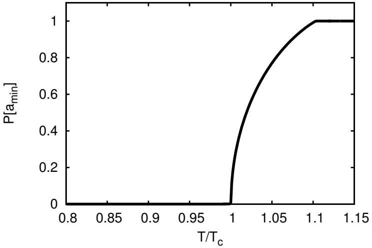
We have mainly two way to address non-vanishing temperatures:

- grand canonical ensemble (the straightforward way)
- Polyakov loop potential (the intriguing way)

Polyakov loop potential

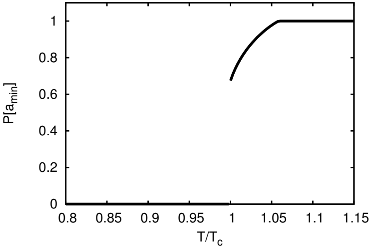
Vacuum Correlators as Ingredients for Finite Temperature Calculations

$T_c \simeq 269$ MeV



SU(2)

$T_c \simeq 283$ MeV



SU(3)

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Fermion sector

The quark-gluon kernel

Ansatz for the quark-gluon kernel

In the exponent of the wave functional

$$S[A,\xi,\xi^\dagger] = \frac{1}{2}\gamma_2 A^2 + \frac{1}{3!}\gamma_3 A^3 + \frac{1}{4!}\gamma_4 A^4 + \xi^\dagger(\bar{\gamma} + \bar{\Gamma}_0 A)\xi$$

take the most simple Dirac and colour structure

$$\bar{\Gamma}_0 \sim \alpha_i\, t^a\, v(\mathbf{p},\mathbf{q})$$

with v being a scalar variational kernel.

Fermion sector

The quark gap equation

Variation of the energy density fixes the quark-gluon vector kernel, which allows to write the quark gap equation

$$M(\mathbf{p}) = \frac{g^2 C_F}{2} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{F(\mathbf{p}-\mathbf{q})}{\mathcal{E}(\mathbf{q})} \left[M(\mathbf{q}) - \frac{\mathbf{p}\cdot\mathbf{q}}{\mathbf{p}^2} M(\mathbf{p}) \right] + \frac{g^2 C_F}{2} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{X(\mathbf{p},\mathbf{q}) v^2(\mathbf{p},\mathbf{q})}{\Omega(\mathbf{p}+\mathbf{q})\mathcal{E}(\mathbf{q})} \mathcal{I}[M,\mathcal{E},\Omega]$$

Looks harmless, but

- bad, bad linear divergences
- chiral condensate and constituent quark mass way too small

Are we missing something?

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Vector kernel with non-trivial Dirac component

Possible Dirac structures

$$\alpha_i \qquad \beta\alpha_i \qquad \gamma_5\alpha_i \qquad \ldots$$

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$$M(0) \simeq 240\,\text{MeV} \qquad \langle\bar{q}q\rangle \simeq (-200\,\text{MeV})^3$$

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Conclusions

Summary

- standard DSE techniques can be used to treat arbitrary wave functionals
- coupled quark-gluon system in Hamiltonian approach investigated
- spurious divergences seem to be now under control

Outlook

- include all perturbatively relevant terms in the Ansatz
- extend the finite-temperature calculations to finite chemical potentials